Quality of High Frequency Measurements: Practical Examples, <u>Theoretical Foundations</u>, and Successful Techniques that Work Past the 40GHz Realm

Samtec-Simberian-Teraspeed Tutorial for DesignCon2010

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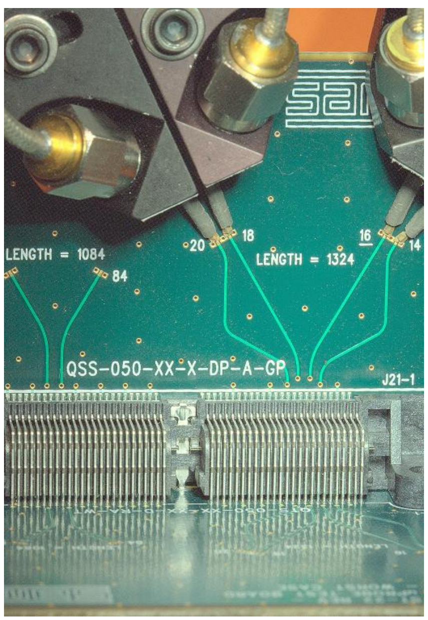

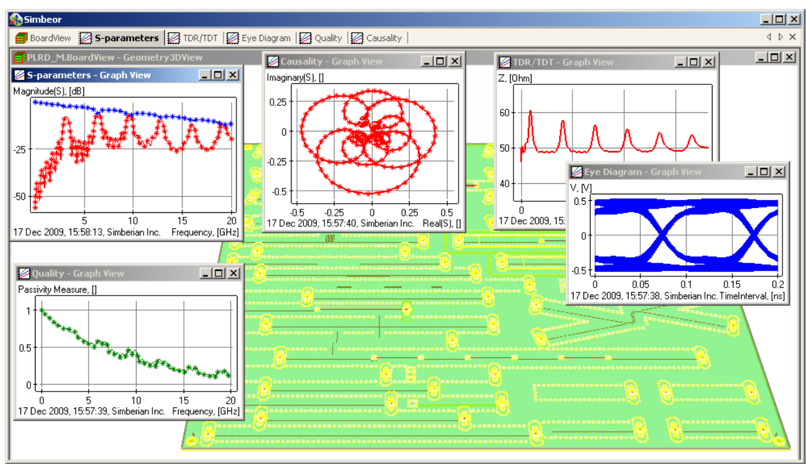

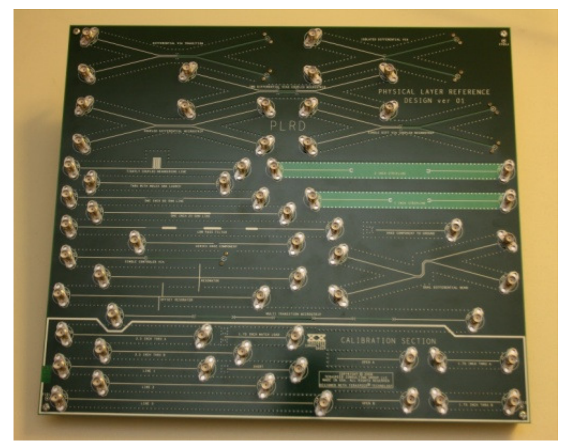

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Outline

- Introduction
- Matrices
- Multiport characterization in frequency-domain
- Multiport characterization in time-domain
- Rational macro-models as the common base
- Global quality metrics in frequency domain
- Practical examples

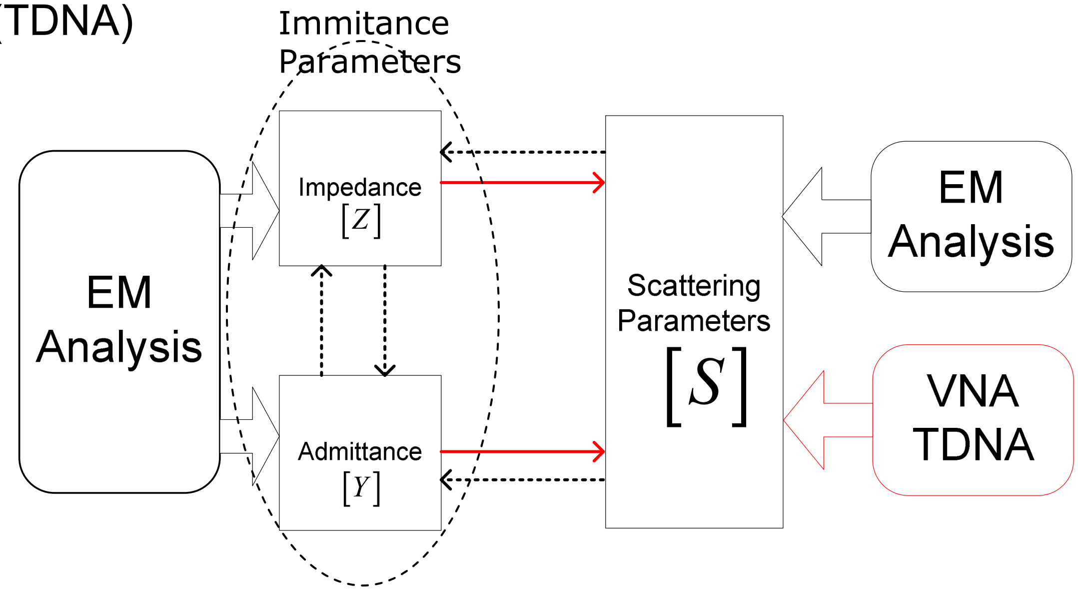
Introduction

- Spectrum of multi-gigabit digital signals falls into the microwave frequency range (1-40 GHz)
- Microwave theory has to be used for analysis of interconnects
- Interconnects can be described as multiports
- Multiport parameters can be computed or measured
- Computed and measured multiport parameters are usually band-limited and may be defective – we need to distinguish good from bad

- **The goal of this tutorial is to:**
  - Cover some basics of the multiport theory
  - Introduce quality metrics for multiport parameters and illustrate it with practical examples

Multiport parameters in general

- Multiport is a natural and scalable black-box description of linear time-invariant systems that are smaller, comparable with, or larger than wavelength
- Multiport parameters are typically available as tabulated output of electromagnetic simulators as well as Vector Network Analyzers (VNA) and Time-Domain Network Analyzers (TDNA)



- Multiport parameters of interconnects have to be reciprocal, passive, causal and corresponding time-domain models must be stable

5 points to learn about multiport parameters

1. Reciprocity property of multiports
2. Passivity of multiports with band-limited response
3. Effect of geometrical symmetry
4. Bandwidth and sampling in frequency domain
5. Multiport macro-models for consistent frequency and time-domain analyses

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Linear algebra

- Linear algebra and complex analysis form the foundation of the multiport theory and help to estimate the multiport parameters goodness
- There are two types of objects in linear algebra: scalars and vectors
  - Scalars – just real or complex numbers
  - Vectors – objects that have direction and magnitude and usually defined by one-dimensional arrays of real or complex numbers or functions
  - Currents, voltages, and waves can be described as scalars or vectors
- Matrices - linear transformations of vectors are usually defined by two-dimensional arrays of real or complex numbers or functions
  - Multiports can be described with impedance, admittance or scattering matrices (descriptors)

See basic definitions in references and backup slides

Some matrix operations

- Transposition – rows become columns

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix}, \; A \in C^{4\times 4} \quad \Rightarrow \quad A^t = \begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} & A_{4,1} \\ A_{1,2} & A_{2,2} & A_{3,2} & A_{4,2} \\ A_{1,3} & A_{2,3} & A_{3,3} & A_{4,3} \\ A_{1,4} & A_{2,4} & A_{3,4} & A_{4,4} \end{bmatrix}$$

- Hermitian-adjoint (Hermitian-conjugate)

$$A^* = \begin{bmatrix} A^*_{1,1} & A^*_{2,1} & A^*_{3,1} & A^*_{4,1} \\ A^*_{1,2} & A^*_{2,2} & A^*_{3,2} & A^*_{4,2} \\ A^*_{1,3} & A^*_{2,3} & A^*_{3,3} & A^*_{4,3} \\ A^*_{1,4} & A^*_{2,4} & A^*_{3,4} & A^*_{4,4} \end{bmatrix}$$

complex-conjugate and transposed

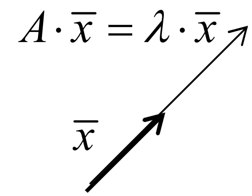
- Symmetric matrix: $A = A^t \;\; or \;\; A_{i,j} = A_{j,i}$
- Hermitian matrix (self-adjoint): $A = A^* \;\; or \;\; A_{i,j} = A^*_{j,i}$
- Transposition of a product: $(A \cdot B)^t = B^t \cdot A^t$
- Hermitian-adjoint of a product: $(A \cdot B)^* = B^* \cdot A^*$
- Inversion of product: $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$
- Two N by N matrices do not commute in general: $A \cdot B \neq B \cdot A$

Eigenvalues and singular values

- An *eigenvalue and eigenvector of a square matrix A are a scalar* λ *and a nonzero* vector *x so that*

$$A \cdot \overline{x} = \lambda \cdot \overline{x}$$

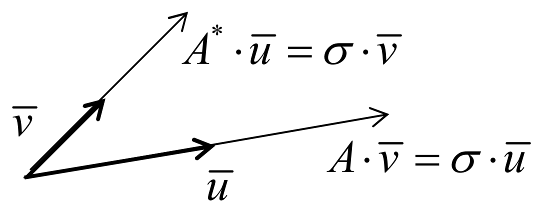
$$\left(A - \lambda \cdot I\right) \cdot \overline{x} = 0 \qquad \det\left(A - \lambda \cdot I\right) = 0$$ - characteristic polynomial



matrix characteristic values

- A *singular value and pair of singular vectors of a square or rectangular matrix A* are a nonnegative scalar σ *and two nonzero vectors u and v so that*

$$A \cdot \overline{v} = \sigma \cdot \overline{u}$$
$$A^{*} \cdot \overline{u} = \sigma \cdot \overline{v}$$



$$\det\left(A \cdot A^{*} - \sigma^{2} \cdot I\right) = 0$$ - a polynomial to find singular values

distance to zero matrix

Decomposition and diagonalization

- Eigenvalue decomposition (only if all eigenvectors are linearly independent)

$$\left.\begin{array}{l} A\cdot\overline{x}_i=\lambda_i\cdot\overline{x}_i,\quad A\in C^{N\times N} \\ X=\left[\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_N\right] \\ \Lambda=diag\left[\lambda_1,\lambda_2,\ldots,\lambda_N\right] \end{array}\right\} A\cdot X=X\cdot\Lambda \Rightarrow$$

$$A=X\cdot\Lambda\cdot X^{-1} \quad \text{(EVD)}$$

diagonal form: $\Lambda=X^{-1}\cdot A\cdot X$

- Singular value decomposition (exists always)

$$\left.\begin{array}{l} A\cdot\overline{v}_n=\sigma_n\cdot\overline{u}_n,\quad A\in C^{N\times M} \\ A^*\cdot\overline{u}_m=\sigma_m\cdot\overline{v}_m \\ U=\left[\overline{u}_1,\overline{u}_2,\ldots,\overline{u}_M\right] \quad U^*\cdot U=I \\ V=\left[\overline{v}_1,\overline{v}_2,\ldots,\overline{v}_N\right] \quad V^*\cdot V=I \\ \Sigma=diag\left[\sigma_1,\sigma_2,\ldots,\sigma_{\min(N,M)}\right] \end{array}\right\} \begin{array}{l} A\cdot V=U\cdot\Sigma \\ A^*\cdot U=V\cdot\Sigma^t \end{array} \Rightarrow$$

$$A=U\cdot\Sigma\cdot V^* \quad \text{(SVD)}$$

diagonal form: $\Sigma=U^*\cdot A\cdot V$

Formulas useful for passivity concept

- Hermitian matrix diagonalization with unitary matrix

$$H = X \cdot \Lambda \cdot X^*, \quad X^* \cdot X = I$$ eigenvalues are real

- Matrices $A \cdot A^*$ and $A^* \cdot A$ are Hermitian

- Eigen-values of $A \cdot A^*$ and singular values of matrix *A*

$$A^* \cdot A = V \cdot \Sigma^* \cdot U^* \cdot U \cdot \Sigma \cdot V^* = \underbrace{V \cdot \Sigma^t \cdot \Sigma \cdot V^* = V \cdot \Lambda \cdot V^*}$$

Eigenvalues are real positive and equal to squares of singular values

- Singular values of symmetric matrix are equal to magnitudes of eigenvalues

Quadratic forms (energy)

- Quadratic form is a homogeneous polynomial of degree two in a number of variables – example with two real variables:

$$q(x,y) = a \cdot x^2 + b \cdot x \cdot y + c \cdot y^2 \quad \text{or} \quad q(x,y) = [x,y] \cdot \begin{bmatrix} a & 0.5 \cdot b \\ 0.5 \cdot b & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- For complex variables in general:

$$q(\overline{x}) = \sum_{i,j=1}^{N} H_{i,j} \cdot x_i^* \cdot x_j = \overline{x}^* \cdot H \cdot \overline{x}$$

If *H* is Hermitian matrix it can be diagonalized as:

$$\Lambda = X^* \cdot H \cdot X, \quad X^* \cdot X = I$$

In new basis, the quadratic form becomes diagonal:

$$q(\overline{x}) = \overline{x}^* \cdot X^* \cdot \Lambda \cdot X \cdot \overline{x} = \sum_{i,j=1}^{N} \lambda_i \cdot \left| x_i' \right|^2$$

It is not negative for all x if eigenvalues of H are not negative!

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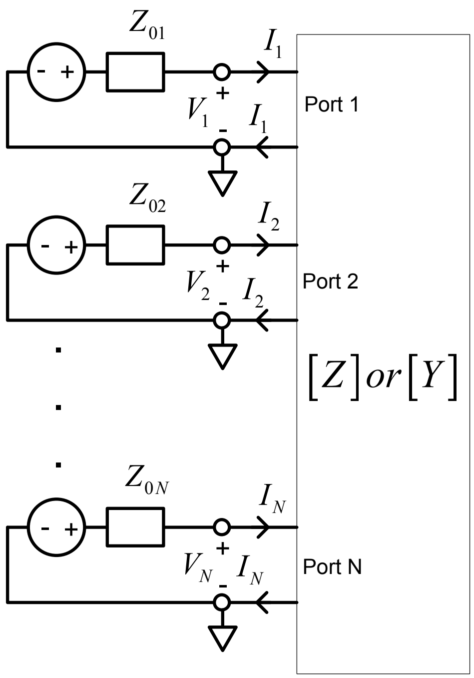
Impedance and admittance parameters



Equivalent currents and voltages at ports:

$$\bar{I} \in C^{N\times 1},\ \bar{V} \in C^{N\times 1},\ \bar{I} = \left(I_1, I_2, ..., I_N\right)^t,\ \bar{V} = \left(V_1, V_2, ..., V_N\right)^t$$

Impedance parameters: $\bar{V} = Z \cdot \bar{I}, \quad Z \in C^{N\times N}$

Admittance parameters: $\bar{I} = Y \cdot \bar{V}, \quad Y \in C^{N\times N}$

Conversion: $Y = Z^{-1}, \qquad Z = Y^{-1}$

$C^{N\times 1}$ space of column-vectors with N complex elements

$C^{N\times N}$ space of complex NxN matrices

- Matrix elements may have large dynamic range
- Difficult to measure directly at high frequencies (difficult to re-calculate I and V from one location to another)
- Convenient for analysis of circuits and power distribution systems (PDNs)

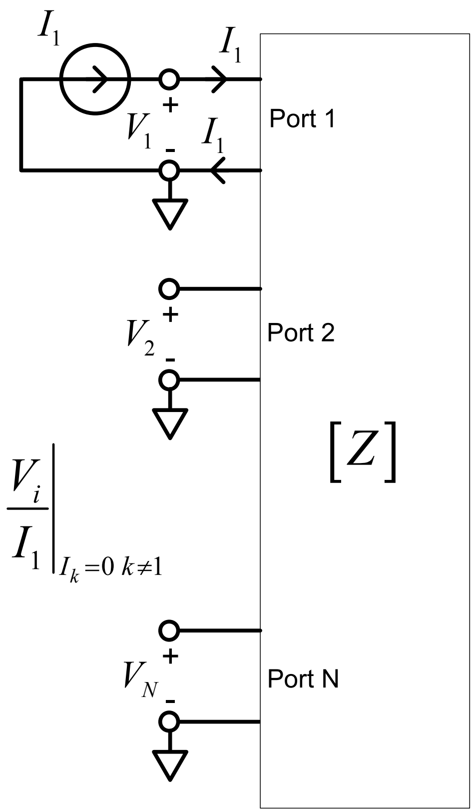
Finding columns of impedance matrix

Z is “open-circuit” matrix

To find elements of a column 1 for instance:

1.Connect current source to port 1 and leave all other ports open-circuited (zero current)
2.Measure voltages at all ports
3.The ratios of voltages and current at port 1 produces one column of matrix Z
4.Repeat for all ports to fill Z

$$Z_{i,1} = \left.\frac{V_i}{I_1}\right|_{I_k=0\ k\neq 1}$$



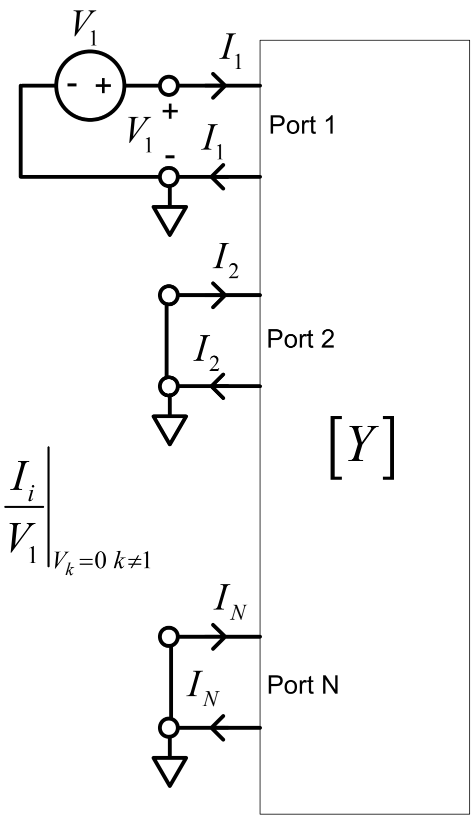
All possible voltages are defined by linear combination of columns of Z (they are in the column-space of Z)

Finding columns of admittance matrix

Y is “short-circuit” matrix

To find elements of a column 1 for instance:
1. Connect voltage source to port 1 and leave all other ports short-circuited (zero voltage)
2. Measure currents at all ports
3. The ratios of currents and voltage at port 1 produces one column of matrix Y
4. Repeat for all ports to fill Y

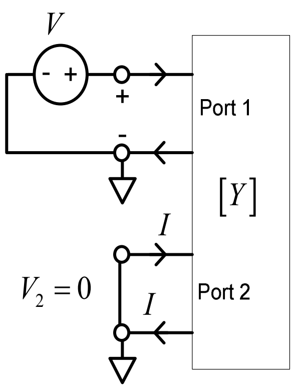
$$Y_{i,1} = \left.\frac{I_i}{V_1}\right|_{V_k=0\ k\neq 1}$$



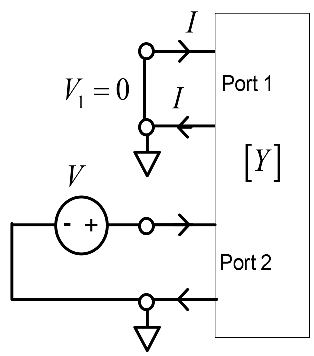
All possible currents are defined by linear combination of columns of Y (they are in the column-space of Y)

Reciprocity

- Linear circuits with reciprocal materials are reciprocal according to Lorentz's theorem of reciprocity:
  Current observed at port 2 with voltage source at port 1 is equal to current observed at port 1 with the same voltage source at port 2 (same with current sources and observed voltages)



$$I = Y_{2,1} \cdot V$$



$$I = Y_{1,2} \cdot V \quad \Rightarrow \quad Y_{2,1} = Y_{1,2}$$

- In general it means that the admittance and impedance matrices are symmetric:

$$Y_{i,j} = Y_{j,i} \;\; or \;\; Y = Y^t \qquad Z_{i,j} = Z_{j,i} \;\; or \;\; Z = Z^t$$ for all frequencies

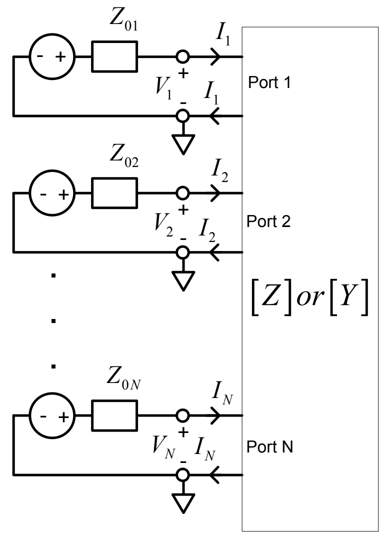
Passivity

- Power transmitted to multiport

$$P_{in} = \mathrm{Re}\left[\bar{I}^* \cdot \bar{V}\right] = \frac{1}{2}\left[\bar{I}^* \cdot \bar{V} + \bar{V}^* \cdot \bar{I}\right]$$

$$P_{in} = \frac{1}{2}\left[\bar{I}^* \cdot \left[Z + Z^*\right] \cdot \bar{I}\right] = \frac{1}{2}\left[\bar{V}^* \cdot \left[Y + Y^*\right] \cdot \bar{V}\right]$$

must be positive for passive structures
(no energy generated for any V or I)



- Hermitian quadratic form is non-negative (Golub & Van Loan):

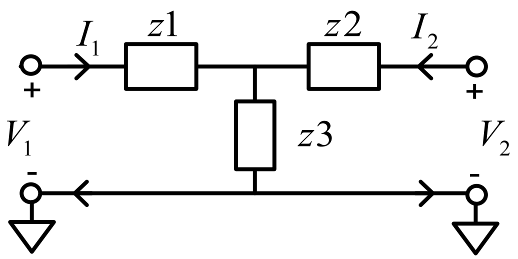
$$iff\ \ eigenvals\left[Z + Z^*\right] \geq 0 \quad \text{or} \quad iff\ \ eigenvals\left[Y + Y^*\right] \geq 0$$

- Reciprocal systems with symmetric matrices:

$$iff\ \ eigenvals\left[\mathrm{Re}\left(Z\right)\right] \geq 0 \quad \text{or} \quad iff\ \ eigenvals\left[\mathrm{Re}\left(Y\right)\right] \geq 0$$

Conditions are sufficient if verified at all frequencies (from DC to infinity)

Example: T-circuit, two-port

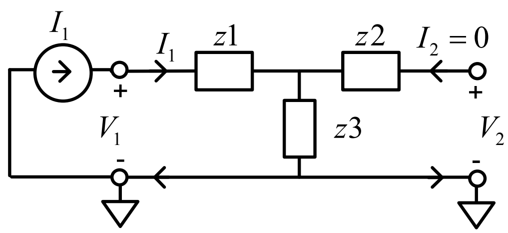


z1, z2, z3 are complex impedances

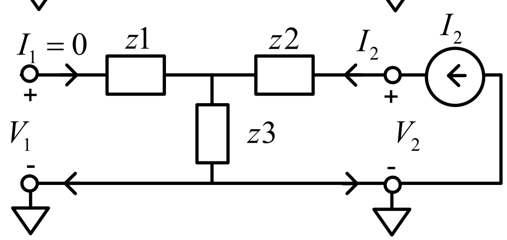
Reciprocal

$$Z = \begin{bmatrix} z1 + z3 & z3 \\ z3 & z2 + z3 \end{bmatrix}, \quad Z \in C^{2\times 2}$$

Passivity: $eigenvals\left[\mathrm{Re}(Z)\right] \geq 0$



$$Z_{1,1} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z1 + z3 \qquad Z_{2,1} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z3$$



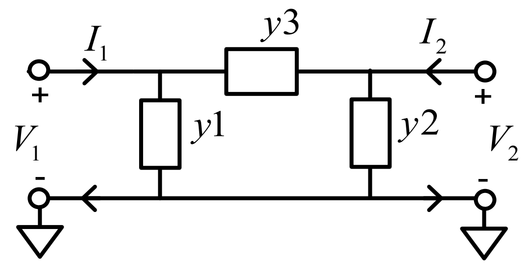
$$Z_{2,2} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z2 + z3 \qquad Z_{1,2} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z3$$

$$Y = Z^{-1} = \frac{1}{z1 \cdot z2 + z1 \cdot z3 + z2 \cdot z3} \begin{bmatrix} z2 + z3 & -z3 \\ -z3 & z1 + z3 \end{bmatrix}$$

Both Z and Y are always symmetric (reciprocal)!

Passivity: $eigenvals\left[\mathrm{Re}(Y)\right] \geq 0$ Always satisfied for nets composed of R,L,C

Example: PI-circuit, two-port

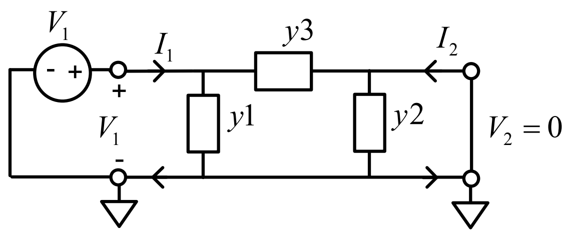


y1, y2, y3 are complex admittances

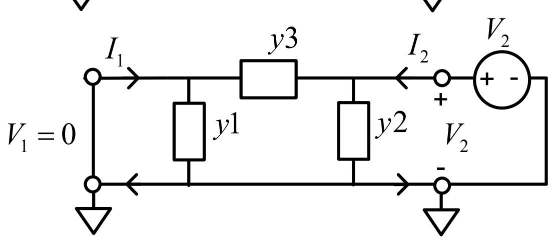
Reciprocal

$$Y = \begin{bmatrix} y1 + y3 & -y3 \\ -y3 & y2 + y3 \end{bmatrix}, \quad Y \in C^{2\times 2}$$

Passivity: $eigenvals\left[\mathrm{Re}\left(Y\right)\right] \geq 0$



$$Y_{1,1} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = y1 + y3 \qquad Y_{2,1} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -y3$$



$$Y_{2,2} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = y2 + y3 \qquad Y_{1,2} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -y3$$

$$Z = Y^{-1} = \frac{1}{y1 \cdot y2 + y1 \cdot y3 + y2 \cdot y3}\begin{bmatrix} y2 + y3 & y3 \\ y3 & y1 + y3 \end{bmatrix}$$

Both Z and Y are always symmetric (reciprocal)!

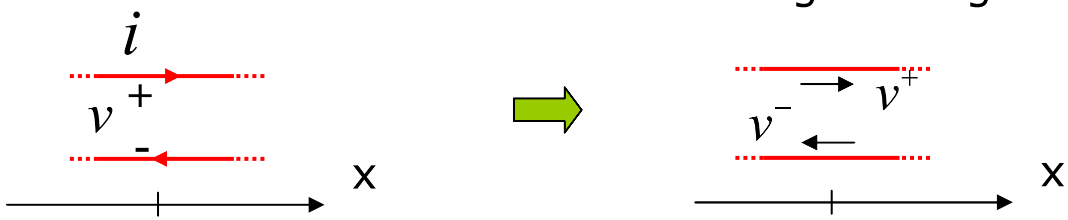
Passivity: $eigenvals\left[\mathrm{Re}\left(Z\right)\right] \geq 0$ Always satisfied for nets composed of R,L,C

Waves in transmission lines

Voltage or current waves are solutions of homogeneous Telegrapher's equations both in frequency and time domain

Current and voltage at location x along the line

can be expressed through voltage waves



$$v(x) = v^{+} \cdot \exp(-\Gamma \cdot x) + v^{-} \cdot \exp(\Gamma \cdot x)$$

$$i(x) = \frac{1}{Z_0}\left[v^{+} \cdot \exp(-\Gamma \cdot x) - v^{-} \cdot \exp(\Gamma \cdot x) \right]$$

$$Z_0(\omega) = \sqrt{z(\omega)/y(\omega)}$$

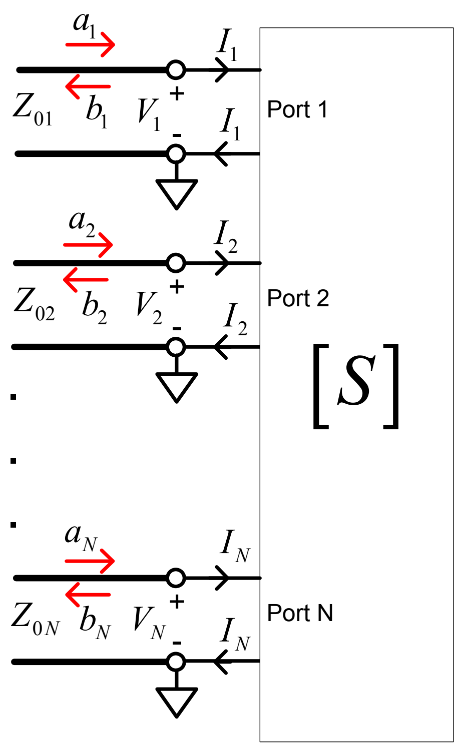
$$\Gamma(\omega) = \sqrt{z(\omega) \cdot y(\omega)}$$

Complex characteristic impedance and propagation constant can be computed with per unit length impedance (z) and admittance (y) of t-line

See generalized transmission line theory in references and backup slides

Scattering parameters definition through transmission line waves

Waves in transmission lines



Instead of voltages and currents in t-lines:

$$V_n = \left[v_n^+ \cdot \exp(\Gamma_n \cdot x) + v_n^- \cdot \exp(\Gamma_n \cdot x) \right]_{x=0} = v_n^+ + v_n^-$$

$$I_n = \frac{1}{Z_{0n}} \left[v_n^+ \cdot \exp(-\Gamma_n \cdot x) - v_n^- \cdot \exp(\Gamma_n \cdot x) \right]_{x=0} = \frac{1}{Z_{0n}} \left[v_n^+ - v_n^- \right]$$

New variables can be introduced by scaling of voltage waves (normalization):

Incident and reflected waves

$$a_n = \frac{1}{\sqrt{Z_{0n}}} \cdot v_n^+, \quad b_n = \frac{1}{\sqrt{Z_{0n}}} \cdot v_n^-$$

Power transmitted by incident and reflected waves:

$$P_n^+ = \frac{\left(v_n^+\right)^2}{Z_{0n}} = |a_n|^2, \quad P_n^- = \frac{\left(v_n^-\right)^2}{Z_{0n}} = |b_n|^2$$

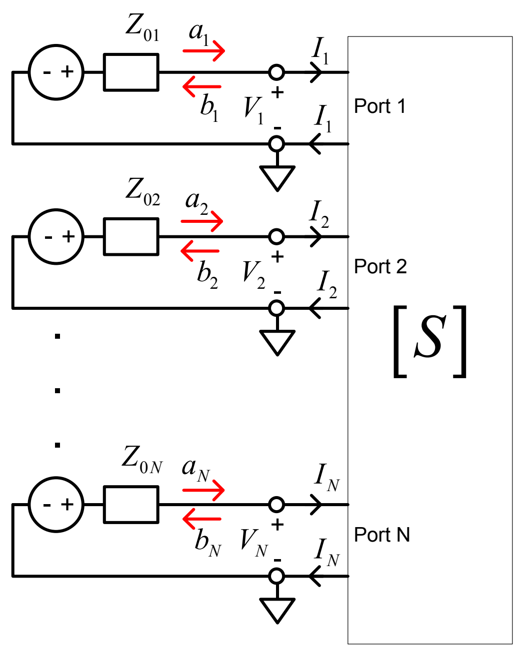
Incident and reflected wave vectors:

$$\overline{a} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\overline{V} + Z_0 \cdot \overline{I} \right), \quad \overline{b} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\overline{V} - Z_0 \cdot \overline{I} \right), \quad \overline{a}, \overline{b} \in C^{N \times 1}$$

$$Z_0 = diag\{Z_{0i}, i = 1, \ldots, N\} \in C^{N \times N}$$

are related by scattering matrix $\quad \overline{b} = S \cdot \overline{a}, \ S \in C^{N \times N}$

Scattering parameters formal definition



Incident and reflected waves may be defined formally:

$$\bar{a} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} + Z_0 \cdot \bar{I} \right), \quad \bar{b} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} - Z_0 \cdot \bar{I} \right), \quad \bar{a}, \bar{b} \in C^{N\times 1}$$

$Z_0 = diag\{Z_{0i}, i = 1, ..., N\} \in C^{N\times N}$ normalization impedances

Scattering matrix defines reflected waves for any incident:

$$\bar{b} = S \cdot \bar{a}, \qquad S \in C^{N\times N}$$

S-matrix can be expressed through admittance or impedance matrices (Cayley transforms):

$$\left.\begin{aligned} &\bar{I} = Y \cdot \bar{V} \\ &\bar{V} = Z \cdot \bar{I} \\ &\bar{a} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} + Z_0 \cdot \bar{I} \right) \\ &\bar{b} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} - Z_0 \cdot \bar{I} \right) \end{aligned}\right\} \Rightarrow$$

$$S = \left(U - Y_N \right) \cdot \left(U + Y_N \right)^{-1}, \quad Y_N = Z_0^{1/2} \cdot Y \cdot Z_0^{1/2}$$

$$S = \left(Z_N - U \right) \cdot \left(U + Z_N \right)^{-1}, \quad Z_N = Z_0^{-1/2} \cdot Z \cdot Z_0^{-1/2}$$

normalized immitance parameters

U is unit matrix here – units on diagonal and other elements are zeroes

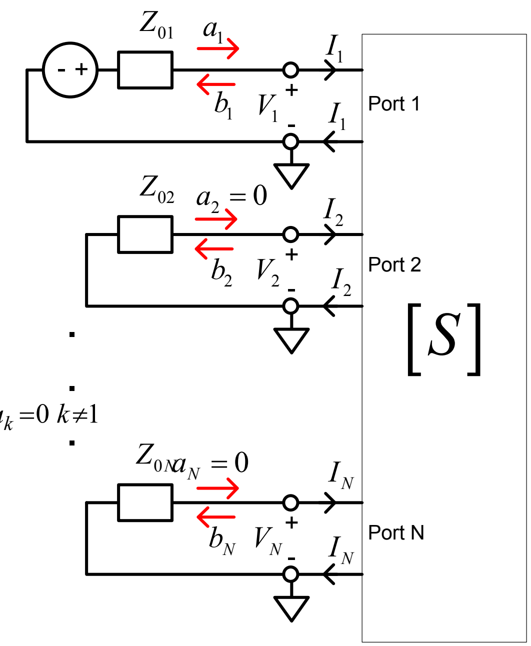
Finding columns of S-matrix

S is the matrix of scattering parameters

To find elements of a column 1 for instance:

1. Launch incident wave into port 1 and terminate all ports with the normalization impedances
2. Measure reflected waves at all ports
3. The ratios of reflected waves and incident wave at port 1 produces one column of matrix S
4. Repeat for all ports to fill S

$$S_{i,1} = \left.\frac{b_i}{a_1}\right|_{a_k=0\ k\neq 1}$$

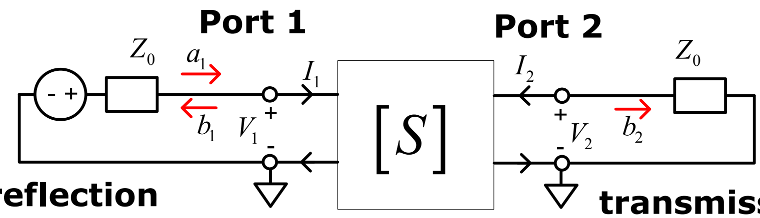


All possible reflected waves are defined by linear combination of columns of S (they are in the column-space of S)

Waves can be measured at any location along line and re-computed to the ports!

S-parameters definitions for 2-port model

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



reflection parameter

$$S_{1,1} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

transmission parameter

$$S_{2,1} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

$$v_i^+ = \sqrt{Z_0} \cdot a_i$$ **voltage of incident wave**

$$v_i^- = \sqrt{Z_0} \cdot b_i$$ **voltage of reflected wave**

$$V_i = v_i^+ + v_i^-$$ **total voltage**

$$I_i = \frac{1}{Z_0}\left(v_i^+ - v_i^-\right)$$ **total current**

$$P_i^+ = \left|a_i\right|^2$$ **power of incident wave**

$$P_i^- = \left|b_i\right|^2$$ **power of reflected wave**

$$\left|S_{1,1}\right|^2 = \frac{\left|b_1\right|^2}{\left|a_1\right|^2} = \frac{P_1^-}{P_1^+} \qquad \left|S_{2,1}\right|^2 = \frac{\left|b_2\right|^2}{\left|a_1\right|^2} = \frac{P_2^-}{P_1^+}$$

Magnitude is limited by 1 for passive systems!

$$\left|S_{i,j}\right| = \sqrt{\mathrm{Re}\left(S_{i,j}\right)^2 + \mathrm{Im}\left(S_{i,j}\right)^2}$$ **magnitude**

$$\angle S_{i,j} = \arctan\left(\mathrm{Im}\left(S_{i,j}\right) / \mathrm{Re}\left(S_{i,j}\right)\right)$$ **phase**

$$\left|S_{i,j}\right|_{dB} = 20 \cdot \log\left(\left|S_{i,j}\right|\right)$$ **magnitude in dB**

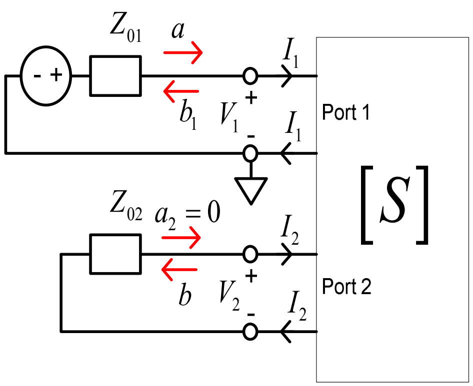
$$i = 1,2; \quad j = 1,2;$$

General properties of S-parameters

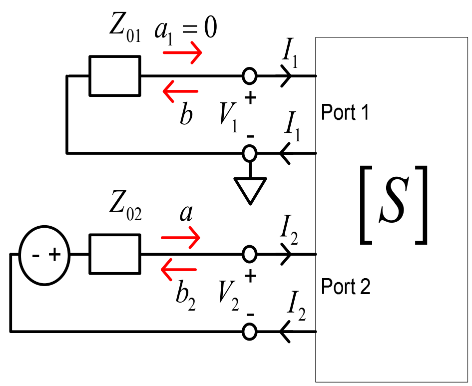
- Any multiport can be characterized with the scattering parameters (even radiation can be included)
- Does not require static definition of current and voltage - waves can be defined as projections on eigen-waves of a wave-guiding structure (wave channels)
- S-parameters can be extracted from electromagnetic simulation or measured at any frequency including DC
- S-parameters are free from singularities and magnitude is bounded by 1 for passive systems
- Easy to create macro-models due to the boundedness

Reciprocity

- Linear circuits with reciprocal materials are reciprocal according to Lorentz's theorem of reciprocity:
  Reflected wave measured at port 2 with incident wave at port 1 is equal to reflected wave measured at port 1 with the same incident wave at port 2



$$b = S_{2,1} \cdot a$$



$$b = S_{1,2} \cdot a \quad \Rightarrow \quad S_{2,1} = S_{1,2}$$

- In general it means that the scattering matrices are symmetric

$$S_{i,j} = S_{j,i} \quad or \quad S = S^t \qquad \text{at all frequencies}$$

Reciprocity estimation and enforcement

Reciprocity measure can be computed as mean difference between elements that have to be equal (at each frequency point):

$$RM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j} - S_{j,i} \right|$$

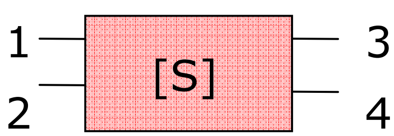
or max singular value of $S - S^t$ can be used

RM is compared with a *threshold*: if *RM* > *threshold*, the multiport is reported as not reciprocal

Averaging can be used to "enforce" the reciprocity (works only with noisy data):

$$S_{j,i} = S_{i,j} = 0.5\left(S_{i,j} + S_{j,i}\right)$$

Example of S-parameters of reciprocal 4-port interconnect (symmetric matrix):

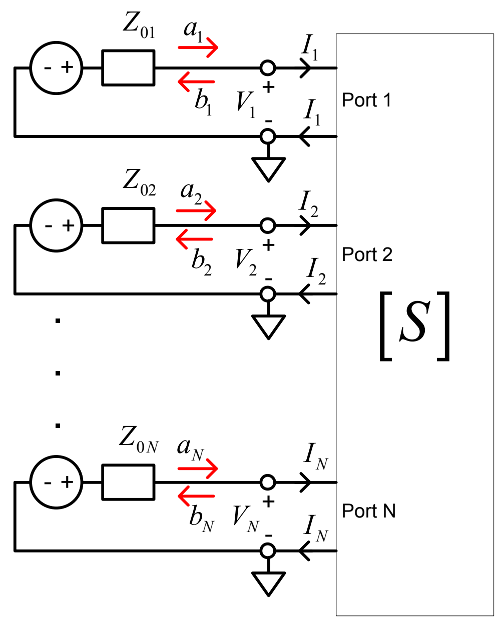


$$S = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \end{bmatrix}$$

RM=0

Passivity

- Power transmitted to multiport is a difference of power transmitted by incident and reflected waves:
$$P_{in} = \sum_{n=1}^{N} |a_n|^2 - |b_n|^2 = \left[\bar{a}^* \cdot \bar{a} - \bar{b}^* \cdot \bar{b}\right]$$
or
$$P_{in} = \bar{a}^* \cdot \bar{a} - \bar{a}^* \cdot S^* S \cdot \bar{a} = \bar{a}^* \cdot \left[U - S^* S\right] \cdot \bar{a}$$
Transmitted power is defined by Hermitian quadratic form and must be not negative for passive multiport for any combination of incident waves



- Quadratic form is non-negative if eigenvalues of the matrix are non-negative (Golub & Van Loan):

$$eigenvals\left[U - S^* \cdot S\right] \ge 0 \;\Rightarrow\; eigenvals\left[S^* \cdot S\right] \le 1$$
(*U* is unit matrix)

Condition is sufficient only if satisfied at all frequencies from DC to infinity

More on passivity

- Maximal singular value of S can be used for passivity estimation:

$$\sigma_{\max} \le 1, \qquad \sigma_i = \sqrt{\lambda_i}, \quad \lambda_i = eigenvals\left(S^* \cdot S\right) \; \lambda_i \in R, \lambda_i \ge 0$$

non-zero singular values of S are square roots of eigenvalues of *S*S* (Golub & Van Loan)

- Passivity of symmetric S can be estimated with eigenvalues as

$$\left| eigenvals\left(S\right) \right| \le 1$$

singular values of symmetric matrices are equal to the magnitudes of the eigenvalues

- Lossless reciprocal system – matrix S is unitary:

$$P_{in} = \bar{a}^* \cdot \left[U - S^* S \right] \cdot \bar{a} = 0 \;\Rightarrow\; S^* S = U \;\Rightarrow\; \sum_{k=1}^{N} \left| S_{i,k} \right|^2 = 1 \quad \sum_{k=1}^{N} S_{i,k}^* \cdot S_{k,j} \Big|_{i \ne j} = 0$$

- Common mistake is to estimate passivity as:

$$\sum_{k=1}^{N} \left| S_{i,k} \right|^2 \le 1 \qquad \text{or} \qquad \left| S_{i,k} \right| \le 1$$

This is necessary but not sufficient condition!

Passivity estimation and enforcement

- Passivity conditions for S-parameters (energy dissipation condition):

$$eigenvals(U - S^* \cdot S) \geq 0 \quad \Rightarrow \quad eigenvals(S^* \cdot S) \leq 1$$

Passivity measure can be computed at each frequency point as:

$$PM = \sqrt{\max\left[eigenvals(S^* \cdot S)\right]}$$

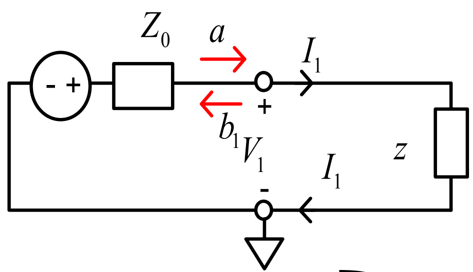
is equal to max singular value of S

PM is compared with a *threshold*: if *PM* > *threshold*, the multiport is reported as not passive

Normalization at each frequency point can used to “enforce” the passivity (works only with minor violations):

$$\begin{aligned} &if \;\; PM > 1.0 \Rightarrow S_p = \frac{S}{PM} \\ &else \;\; S_p = S \end{aligned}$$

Example: Terminator, one-port



z is a complex impedance $S \in C^{1\times 1}$

$$a_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 + Z_0 \cdot I_1\right)$$

$$b_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 - Z_0 \cdot I_1\right)$$

$$V_1 = z \cdot I_1$$

$$b_1 = \frac{z - Z_0}{z + Z_0} \cdot a_1$$

$$S_{1,1} = \frac{z - Z_0}{z + Z_0}$$

Reflection parameter is equal to the reflection coefficient

Short-circuit:

$$z = 0 \Rightarrow S_{1,1} = -1$$

Open-circuit:

$$z = \infty \Rightarrow S_{1,1} = 1$$

Passivity:

$$\left|S_{1,1}\right| \le 1$$

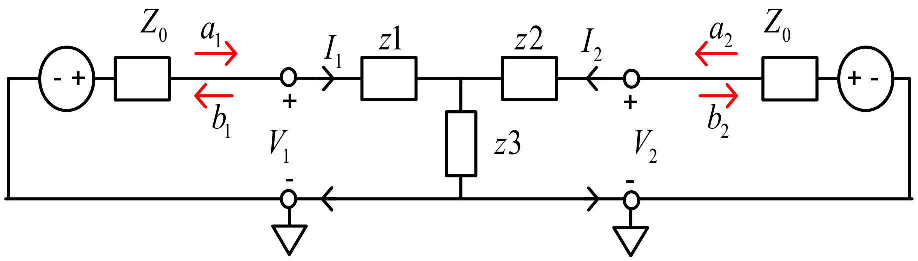
For real normalization impedance $\mathrm{Re}(z) \ge 0$

Always satisfied for nets composed of R,L,C

Alternatively we can transform Z into S with

$$S = \left(Z_N - U\right)\cdot\left(U + Z_N\right)^{-1}, \quad Z_N = Z_0^{-1/2} \cdot Z \cdot Z_0^{-1/2}$$

Example: T-circuit, two-port



z1, z2, z3 are complex impedances

$$S \in C^{2\times 2}$$

We just use known Z and transform it to S

$$Z = \begin{bmatrix} z1+z3 & z3 \\ z3 & z2+z3 \end{bmatrix} \qquad Z_N = \frac{1}{Z_0}\begin{bmatrix} z1+z3 & z3 \\ z3 & z2+z3 \end{bmatrix}$$

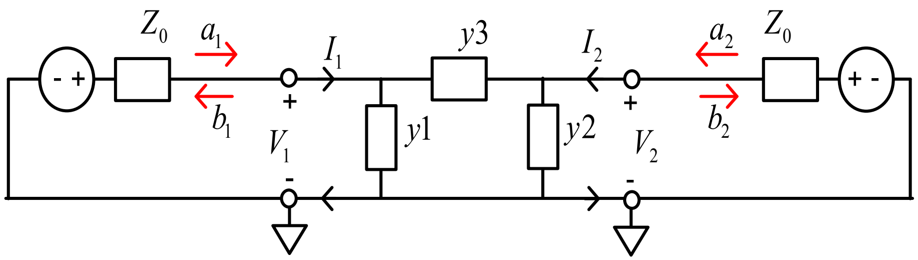
$$S = (Z_N - U)\cdot(U + Z_N)^{-1} = \frac{1}{A}\begin{bmatrix} -Z_0^2 + (z1 - z2)\cdot Z_0 + B & 2\cdot z3\cdot Z_0 \\ 2\cdot z3\cdot Z_0 & -Z_0^2 - (z1-z2)\cdot Z_0 + B \end{bmatrix}$$

$$A = Z_0^2 + (z1 + z2 + 2\cdot z3)\cdot Z_0 + B \qquad B = z1\cdot z2 + z2\cdot z3 + z1\cdot z3$$

S is always symmetric (reciprocal system) and non-singular

Passivity: $\left|eigenvals[S]\right| \le 1$ Always satisfied for nets composed of R,L,C

Example: PI-circuit, two-port



y1, y2, y3 are complex admittances

$$S \in C^{2\times 2}$$

We just use known Z and transform it to S

$$Y = \begin{bmatrix} y1 + y3 & -y3 \\ -y3 & y2 + y3 \end{bmatrix} \quad Y_N = Z_0 \cdot \begin{bmatrix} y1 + y3 & -y3 \\ -y3 & y2 + y3 \end{bmatrix}$$

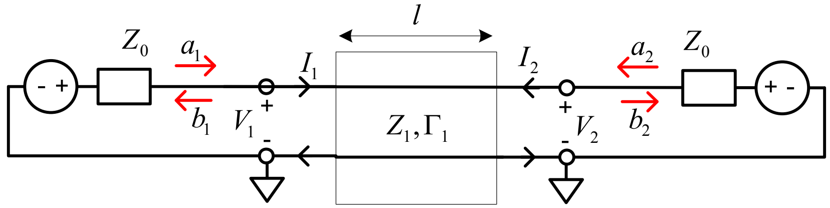
$$S = \left(U - Y_N\right)\cdot\left(U + Y_N\right)^{-1} = \frac{1}{A}\begin{bmatrix} Y_0^2 - \left(y1 - y2\right)\cdot Y_0 - B & 2\cdot y3\cdot Y_0 \\ 2\cdot y3\cdot Y_0 & Y_0^2 + \left(y1 - y2\right)\cdot Y_0 \end{bmatrix} \qquad Y_0 = \frac{1}{Z_0}$$

$$A = Y_0^2 + \left(y1 + y2 + 2\cdot y3\right)\cdot Y_0 + B \quad B = y1\cdot y2 + y2\cdot y3 + y1\cdot y3$$

S is always symmetric (reciprocal system) and non-singular

Passivity: $\left|eigenvals\left[S\right]\right| \le 1$ Always satisfied for nets composed of R,L,C

One-conductor line segment



Characteristic impedance and propagation constant must be causal and positive-real

$$Y(\omega,l)=\frac{1}{Z_1}\begin{bmatrix} cth(\Gamma_1\cdot l) & -csh(\Gamma_1\cdot l) \\ -csh(\Gamma_1\cdot l) & cth(\Gamma_1\cdot l)\end{bmatrix} \Rightarrow Y_N=\frac{Z_0}{Z_1}\begin{bmatrix} cth(\Gamma_1\cdot l) & -csh(\Gamma_1\cdot l) \\ -csh(\Gamma_1\cdot l) & cth(\Gamma_1\cdot l)\end{bmatrix}$$

$$S(\omega,l)=(U-Y_N)\cdot(U+Y_N)^{-1} \Rightarrow S(\omega,l)=\frac{1}{D}\begin{bmatrix} Z_1^2-Z_0^2 & 2\cdot Z_1\cdot Z_0\cdot csh(\Gamma_1\cdot l) \\ 2\cdot Z_1\cdot Z_0\cdot csh(\Gamma_1\cdot l) & Z_1^2-Z_0^2\end{bmatrix}$$

$$S\in C^{2\times 2}$$

$$D=Z_1^2+Z_0^2+2\cdot Z_1\cdot Z_0\cdot cth(\Gamma_1\cdot l)$$

S-matrix is symmetric (S[1,2]=S[2,1]) and skew-symmetric (S[1,1]=S[2,2])

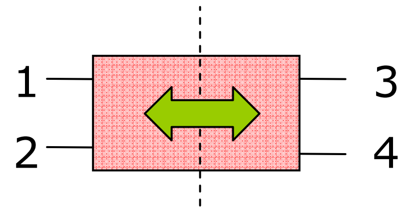
If normalization impedance is equal to the characteristic impedance of the mode, we get generalized modal S-matrix:

$$Z_0=Z_1 \Rightarrow S(\omega,l)=\begin{bmatrix} 0 & \exp(-\Gamma_1\cdot l) \\ \exp(-\Gamma_1\cdot l) & 0\end{bmatrix}$$ (anti-diagonal matrix)

Geometric mirror symmetry input to output

S-matrix of reciprocal 4-port:

$$S=\begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \end{bmatrix}$$



Symmetri group generator:

$$F=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

S-matrix must commute with F: $F\cdot S = S\cdot F \Rightarrow \begin{bmatrix} S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \\ S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \end{bmatrix} = \begin{bmatrix} S_{1,3} & S_{1,4} & S_{1,1} & S_{1,2} \\ S_{2,3} & S_{2,4} & S_{1,2} & S_{2,2} \\ S_{3,3} & S_{3,4} & S_{1,3} & S_{2,3} \\ S_{3,4} & S_{4,4} & S_{1,4} & S_{2,4} \end{bmatrix}$

It means that: $S_{3,3}=S_{1,1},\ S_{2,3}=S_{1,4}$
$S_{4,4}=S_{2,2},\ S_{3,4}=S_{1,2}$

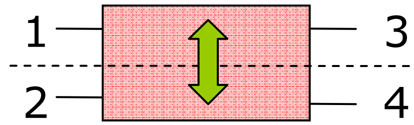
Final S-matrix of reciprocal symmetrical 4-port: $S=\begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{1,4} & S_{2,4} \\ S_{1,3} & S_{1,4} & S_{1,1} & S_{1,2} \\ S_{1,4} & S_{2,4} & S_{1,2} & S_{2,2} \end{bmatrix}$

only 6 independent parameters

Geometric mirror symmetry about the plane along the interconnects (differential nets)

S-matrix of reciprocal 4-port:

$$S = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \end{bmatrix}$$



Symmetry group generator:

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

S-matrix must commute with F: $F \cdot S = S \cdot F \Rightarrow \begin{bmatrix} S_{1,2} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,4} & S_{2,4} & S_{3,4} & S_{4,4} \\ S_{1,3} & S_{2,3} & S_{3,3} & S_{3,4} \end{bmatrix} = \begin{bmatrix} S_{1,2} & S_{1,1} & S_{1,4} & S_{1,3} \\ S_{2,2} & S_{1,2} & S_{2,4} & S_{2,3} \\ S_{2,3} & S_{1,3} & S_{3,4} & S_{3,3} \\ S_{2,4} & S_{1,4} & S_{4,4} & S_{3,4} \end{bmatrix}$

It means that: $S_{2,2} = S_{1,1}, S_{2,3} = S_{1,4}$
$S_{2,4} = S_{1,3}, \; S_{4,4} = S_{3,3}$

Final S-matrix of reciprocal symmetrical 4-port: $S = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{1,2} & S_{1,1} & S_{1,4} & S_{1,3} \\ S_{1,3} & S_{1,4} & S_{3,3} & S_{3,4} \\ S_{1,4} & S_{1,3} & S_{3,4} & S_{3,3} \end{bmatrix}$

only 6 independent parameters

See more on that in Simberian App Note#2009_01: Practical notes on mixed-mode transformations in differential interconnects (with experimental validation), 2009, http://www.simberian.com/AppNotes.php

Geometric symmetry estimation and enforcement

Symmetry measure can be computed as mean difference between elements that have to be equal (at each frequency point):

$$GSM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j} - S_{is,js} \right|$$ or max singular value of $F \cdot S - S \cdot F$ can be used

GSM is compared with a *threshold*: if *GSM* > *threshold*, the multiport is reported as not symmetric

Averaging is used to “enforce” the geometric symmetry (works only for minor violation of symmetry):

$$S_{is,js} = S_{i,j} = 0.5\left(S_{i,j} + S_{is,js}\right)$$

Outline

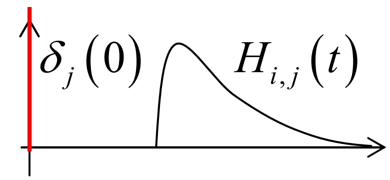
- Introduction
- Matrices
- Multiport characterization in frequency-domain
- Multiport characterization in time-domain
- Rational macro-models as the common base
- Global quality metrics in frequency domain
- Practical examples

Time-domain characterization of multiports

- Time-domain response of a multiport

$$\overline{w}(t) = H(t) * \overline{x}(t) = \int_{-\infty}^{\infty} H(t-\tau)\cdot\overline{x}(\tau)\cdot d\tau, \quad H(t) \in R^{N\times N}$$

Element *[i,j]* of the pulse response matrix *H* is a response at port *i* with port *j* excited with the ideal Dirak pulse (unit-energy pulse)



- Multiport descriptor type defines terminations to find *H*:

$\overline{x}(t) = \overline{I}(t),\ \overline{w}(t) = \overline{V}(t),\ H(t) = Z(t),\ H(i\omega) = Z(i\omega)$ Impedance matrices

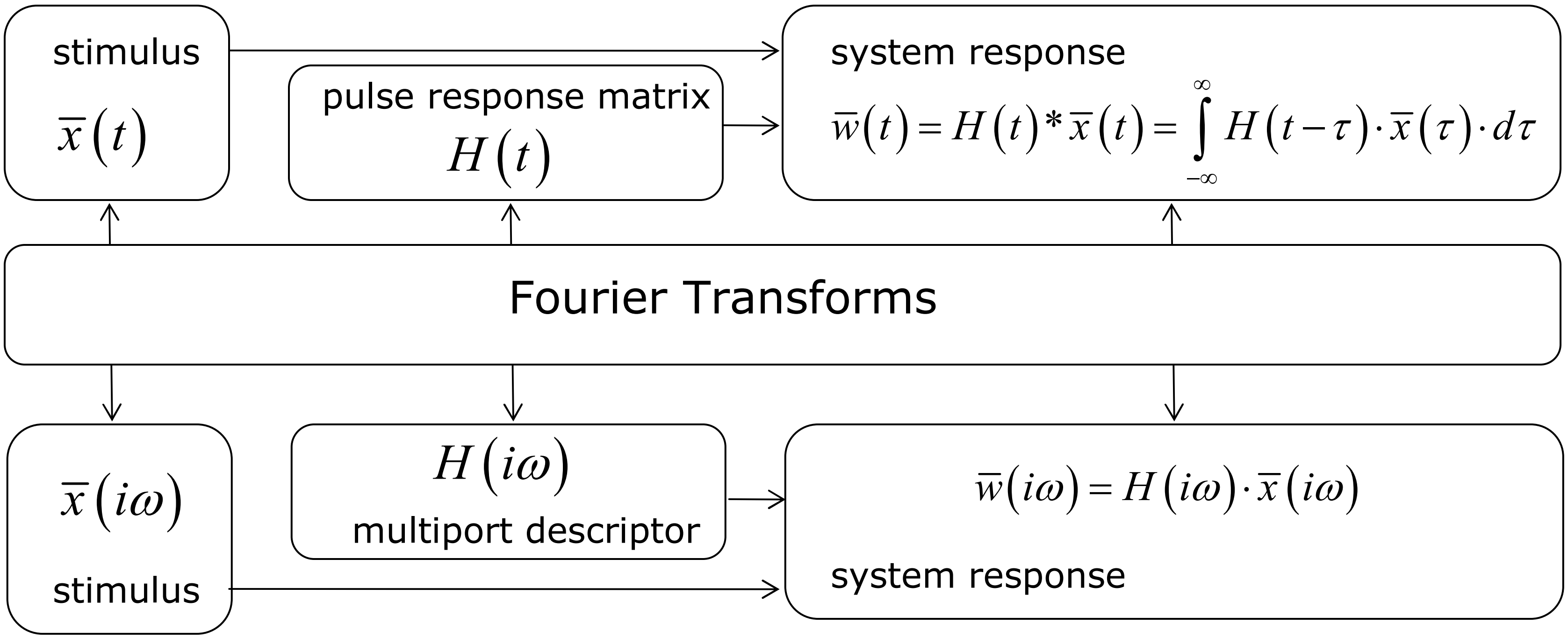
$\overline{x}(t) = \overline{V}(t),\ \overline{w}(t) = \overline{I}(t),\ H(t) = Y(t),\ H(i\omega) = Y(i\omega)$ Admittance matrices

$\overline{x}(t) = \overline{a}(t),\ \overline{w}(t) = \overline{b}(t),\ H(t) = S(t), H(i\omega) = S(i\omega)$ Scattering matrices

- Pulse response matrix and frequency-domain descriptors are related:

$$H(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(i\omega)\cdot e^{i\omega t}\cdot d\omega, \quad H(t) \in R^{N\times N} \quad \Leftrightarrow \quad H(i\omega) = \int_{-\infty}^{\infty} H(t)\cdot e^{-i\omega t}\cdot dt, \quad H(i\omega) \in C^{N\times N}$$

Time and frequency domains for LTI system



$$\bar{x}(t) = \bar{I}(t),\ \bar{w}(t) = \bar{V}(t),\ H(t) = Z(t),\ H(i\omega) = Z(i\omega)$$

$$\bar{x}(t) = \bar{V}(t),\ \bar{w}(t) = \bar{I}(t),\ H(t) = Y(t),\ H(i\omega) = Y(i\omega)$$

$$\bar{x}(t) = \bar{a}(t),\ \bar{w}(t) = \bar{b}(t),\ H(t) = S(t),\ H(i\omega) = S(i\omega)$$

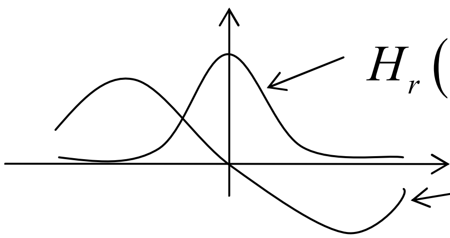
Reference: S.H. Hall, H.L. Heck, Advanced signal integrity for high-speed digital designs, Wiley, 2009, p. 504

Realness of time-domain response

- Time-domain response must be real function of time

$$H(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(i\omega)\cdot e^{i\omega t}\cdot d\omega, \quad H(t)\in R^{N\times N}$$

- It happens if $H(-i\omega) = H^{*}(i\omega)$, or if $H(i\omega) = H_r(\omega) + i\cdot H_i(\omega)$



$H_r(-\omega) = H_r(\omega)$ - even function of frequency

$H_i(-\omega) = -H_i(\omega)$ - odd function of frequency

- Conditions at zero frequency may be useful to restore the DC point:

$$\left.\frac{dH_r(\omega)}{d\omega}\right|_{\omega=0} = 0, \quad H_i(0) = 0$$

DC condition for all multiport parameters

Causality of LTI system

- Time-invariant system does not change behavior with time

$$\bar{x}(t) \to \bar{w}(t) \Rightarrow \bar{x}(t-\tau) \to \bar{w}(t-\tau)$$

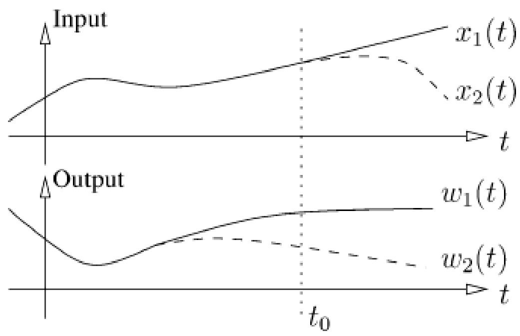
- The system is causal iff any two identical inputs $\bar{x}_1(t) = \bar{x}_2(t), t < t_0$ produce two identical outputs $\bar{w}_1(t) = \bar{w}_2(t), t < t_0$
- The system is causal iff for any input $\bar{x}(t) = 0 \ at \ t < t_0$ the output is $\bar{w}(t) = 0 \ at \ t < t_0$
- The system is causal iff all elements of the time-domain pulse response matrix are

$$H_{i,j}(t) = 0 \ at \ t < 0$$

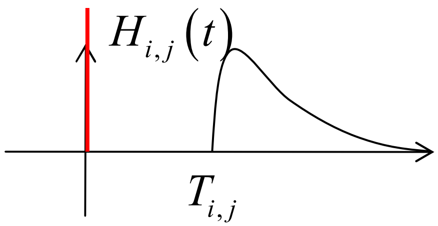
Delayed causality (for interconnects):

$$H_{i,j}(t) = 0 \ at \ t < T_{i,j}, \ T_{i,j} > 0$$

Non-causal response



Delayed-causal pulse response



P. Triverio S. Grivet-Talocia, M.S. Nakhla, F.G. Canavero, R. Achar, Stability, Causality, and Passivity in Electrical Interconnect Models, IEEE Trans. on Advanced Packaging, vol. 30. 2007, N4, p. 795-808.

Stability and passivity in time-domain

- The system is stable if output is bounded for all bounded inputs

$$\left|x(t)\right| < K \Rightarrow \left|w(t)\right| < M, \ \forall t \qquad \text{(BIBO)}$$

- A multiport network is passive if energy absorbed by multiport

$$E(t) = \int_{-\infty}^{t} \overline{V}^t(\tau) \cdot \overline{I}(\tau) \cdot d\tau \geq 0, \ \forall t \qquad \text{(does not generate energy)}$$

for all time and all possible voltages and currents or

$$E(t) = \int_{-\infty}^{t} \left[\overline{a}^t(\tau) \cdot \overline{a}(\tau) - \overline{b}^t(\tau) \cdot \overline{b}(\tau)\right] \cdot d\tau \geq 0, \ \forall t$$

for all possible incident and reflected waves

- If the system is passive according to the above definition, it is also causal

$$\overline{a}(t) = 0, \ \forall t < t_0 \Rightarrow \int_{-\infty}^{t} \left[\overline{b}^t(\tau) \cdot \overline{b}(\tau)\right] \cdot d\tau \leq 0 \Rightarrow \overline{b}(t) = 0, \ \forall t < t_0$$

P. Triverio S. Grivet-Talocia, M.S. Nakhla, F.G. Canavero, R. Achar, Stability, Causality, and Passivity in Electrical Interconnect Models, IEEE Trans. on Advanced Packaging, vol. 30. 2007, N4, p. 795-808.

Causality in frequency-domain

- Condition $H(t)=0\ at\ t<0$ for the time-domain response matrix and

$$H(i\omega)=\int_{-\infty}^{\infty} H(t)\cdot e^{-i\omega t}\cdot dt,\quad H(i\omega)\in C^{N\times N}$$

leads to Hilbert transform or Kramers-Kronig relations between the real and imaginary parts of the frequency-domain parameters

$$H(i\omega)=\frac{1}{i\pi}PV\int_{-\infty}^{\infty}\frac{H(i\omega')}{\omega-\omega'}\cdot d\omega',\quad PV=\lim_{\varepsilon\to 0}\left(\int_{-\infty}^{\omega-\varepsilon}+\int_{\omega+\varepsilon}^{+\infty}\right)$$

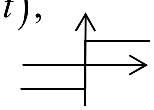
$$H_r(\omega)=\frac{1}{\pi}PV\int_{-\infty}^{\infty}\frac{H_i(\omega')}{\omega-\omega'}\cdot d\omega',\quad H_i(\omega)=\frac{-1}{\pi}PV\int_{-\infty}^{\infty}\frac{H_r(\omega')}{\omega-\omega'}\cdot d\omega'$$

Kramers, H.A., Nature, v 117, 1926 p. 775..
Kronig, R. de L., J. Opt. Soc. Am. N12, 1926, p 547.

Derivation:

$$H(t)=sign(t)\cdot H(t),$$

$$sign(t)=\begin{vmatrix}-1, & t<0\\ 1, & t>0\end{vmatrix}$$



$$H(i\omega)=F\{H(t)\}=$$

$$=\frac{1}{2\pi}F\{sign(t)\}*F\{H(t)\}$$

$$F\{sign(t)\}=\frac{2}{i\omega}$$

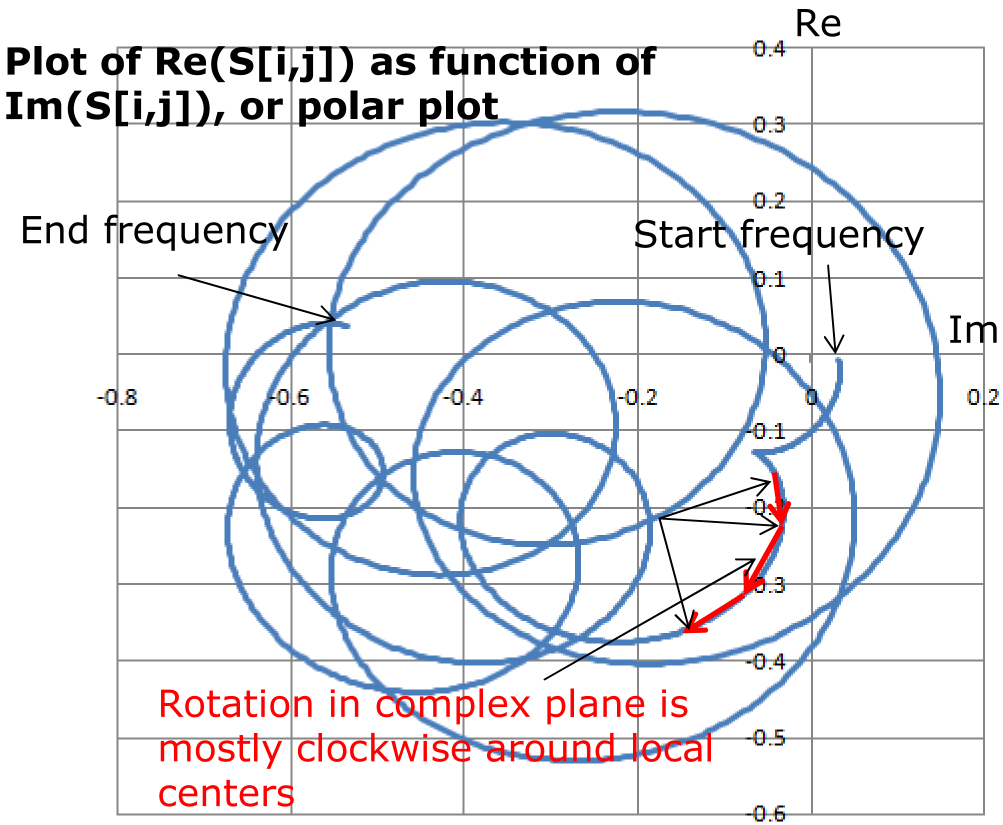
- Real part can be derived from imaginary or vice versa, but it must be known from DC to infinity

Causality estimation - difficult way

- Kramers-Kronig relations cannot be directly checked for the frequency-domain response known over the limited bandwidth
- Causality boundaries can be introduced on the base of the Kramers-Kronig relations to estimate causality of the tabulated and band-limited data sets
  - Milton, G.W., Eyre, D.J. and Mantese, J.V, *Finite Frequency Range Kramers Kronig Relations: Bounds on the Dispersion,* Phys. Rev. Lett. 79, 1997, p. 3062-3064
  - Triverio, P. Grivet-Talocia S., *Robust Causality Characterization via Generalized Dispersion Relations*, IEEE Trans. on Adv. Packaging, N 3, 2008, p. 579-593.

Causality estimation - easy way

- “Heuristic” causality measure based on the observation that polar plot of a causal system rotates mostly clockwise (suggested by V. Dmitriev-Zdorov)



Causality measure (CM) can be computed as the ratio of clockwise rotation measure to total rotation measure in %.

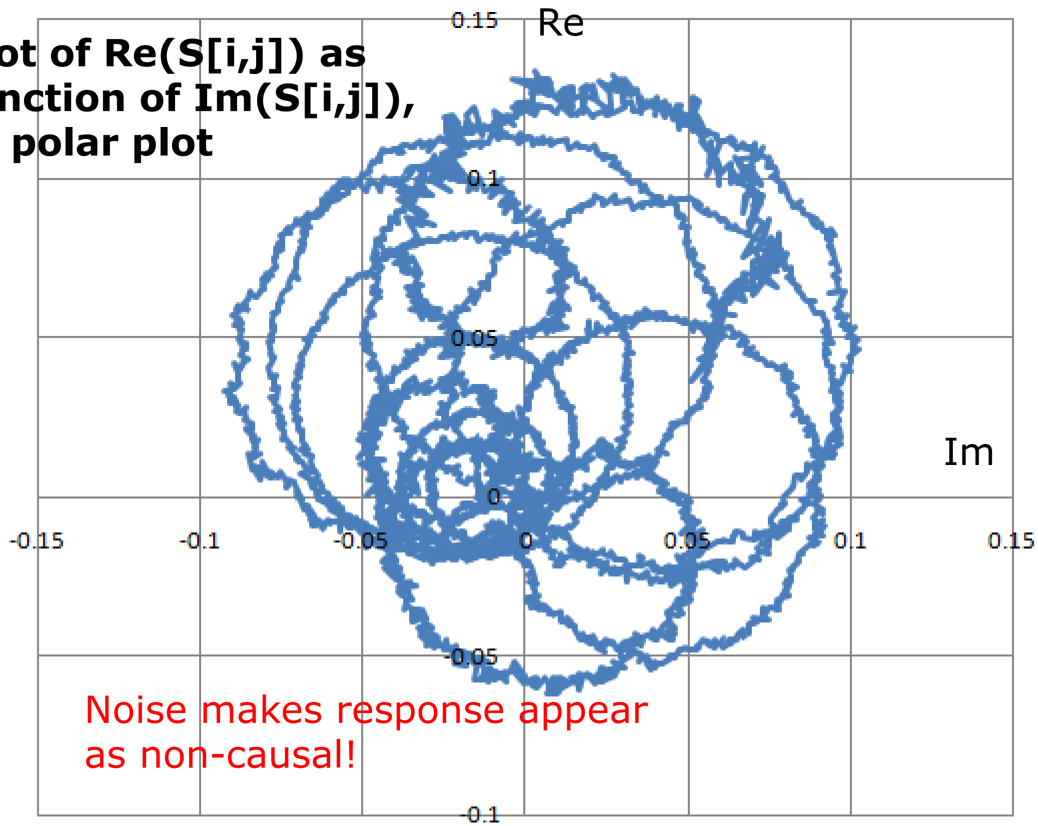
If this value is below 80%, the parameters are reported as suspect for possible violation of causality.

RMS error of rational approximation can be also used as causality measure

Example of non-causal response

- Measured data often exhibit non-causality due to the measurement noise

Plot of Re(S[i,j]) as function of Im(S[i,j]), or polar plot



Noise makes response appear as non-causal!

Filtering or decimating can be used to reduce the noise (see backup slides)

Electromagnetic models with non-causal dielectric models will be reported as causal with “rotation” approach

Causality improvement

- Filtration or decimation – may further degrade response quality

- Artificially extend real or imaginary part, or magnitude of the frequency response to DC and to the infinity and restore the other part with the Kramers-Kronig equations
  - The restored part will strongly depend on the artificial extension
  - Iterative extension adjustment is possible to improve accuracy over the sampled frequency band - difficult to implement

- Fit the response with causal rational basis functions (rational compact model)
  - Provides controlled accuracy over the sampled frequency band
  - Response from DC and to infinity in frequency-domain
  - Consistent results in both frequency and time domains

Outline

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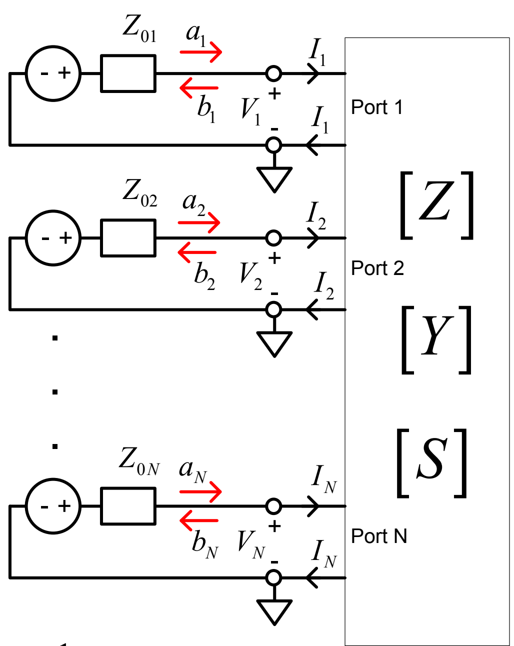
- Global quality metrics in frequency domain
- Practical examples

Rational Compact Models (RCM)



$$\bar{a} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} + Z_0 \cdot \bar{I}\right)$$

$$\bar{b} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V} - Z_0 \cdot \bar{I}\right)$$

Impedance parameters:

$$\bar{V} = Z \cdot \bar{I},\ Z_{i,j} = \left.\frac{V_i}{I_j}\right|_{I_k=0\ k\neq j} \Rightarrow Z_{i,j} = d_{ij} + s \cdot h_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{s - p_{ij,n}} + \frac{r_{ij,n}^*}{s - p_{ij,n}^*} \right)$$

Admittance parameters:

$$\bar{I} = Y \cdot \bar{V},\quad Y_{i,j} = \left.\frac{I_i}{V_j}\right|_{V_k=0\ k\neq j} \Rightarrow Y_{i,j} = d_{ij} + s \cdot h_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{s - p_{ij,n}} + \frac{r_{ij,n}^*}{s - p_{ij,n}^*} \right)$$

Scattering parameters:

$$\bar{b} = S \cdot \bar{a},\quad S_{i,j} = \left.\frac{b_i}{a_j}\right|_{a_k=0\ k\neq j} \Rightarrow S_{i,j} = \left[d_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{s - p_{ij,n}} + \frac{r_{ij,n}^*}{s - p_{ij,n}^*} \right) \right] \cdot e^{-s \cdot T_{ij}}$$

$s = i\omega,\ d_{ij} - values\ at\ \infty,\ h_{ij} - asymptotes,\ N_{ij} - number\ of\ poles,$

$r_{ij,n} - residues,\ p_{ij,n} - poles\ (real\ or\ complex), T_{ij} - optional\ delay$

Properties of RCM for S-parameters

- Pulse response is real and delay-causal

$$S_{i,j}(\omega)=\left[d_{ij}+\sum_{n=1}^{N_{ij}}\left(\frac{r_{ij,n}}{i\omega-p_{ij,n}}+\frac{r_{ij,n}^{*}}{i\omega-p_{ij,n}^{*}}\right)\right]\cdot e^{-i\omega\cdot T_{ij}}$$

$$\Downarrow$$

$$S_{i,j}(t)=0,\ t<T_{ij}$$

$$S_{i,j}(t)=d_{ij}\delta\left(t-T_{ij}\right)+\sum_{n=1}^{N_{ij}}\left[r_{ij,n}\cdot\exp\left(p_{ij,n}\cdot\left(t-T_{ij}\right)\right)+r_{ij,n}^{*}\cdot\exp\left(p_{ij,n}^{*}\cdot\left(t-T_{ij}\right)\right)\right],\ t\geq T_{ij}$$

- Stable $\mathrm{Re}\left(p_{ij,n}\right)<0$
- Passive if $eigenvals\left[S(\omega)\cdot S^{*}(\omega)\right]\leq 1\ \forall\omega,\ from\, 0\ to\, \infty$
- Reciprocal if $S_{i,j}(\omega)=S_{j,i}(\omega)$

May require enforcement

What are RCMs for?

- Improve quality of tabulated Touchstone models
  - Fix minor passivity and causality violations
  - Interpolate and extrapolate with guarantied passivity
- Produce broad-band SPICE models (see backup slides)
  - Much smaller model size
  - No artifacts and guarantied stability of SPICE simulation
  - Consistent frequency and time domain analyses
- Compute time-domain response of a channel with a fast recursive convolution algorithm (exact solution for PWL signals)

Bandwidth and sampling to build RCM

- If no DC point, the lowest frequency in the sweep should be
  - Below the transition to skin-effect (1-50 MHz for PCB applications)
  - Below the first possible resonance in the system (important for cables, L is physical length)

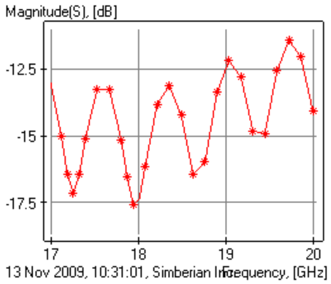
$$L < \frac{\lambda}{4} = \frac{c}{4 f_l \cdot \sqrt{\varepsilon_{eff}}} \Rightarrow f_l < \frac{c}{4L \cdot \sqrt{\varepsilon_{eff}}}$$

- The highest frequency in the sweep must be defined by the required resolution in time-domain or by spectrum of the signal (defined by rise time)

$$f_h > \frac{1}{2t_r}$$

- The sampling is very important for IFFT and convolution-based algorithms, but not so for algorithms based on fitting
  - There must be 3-4 frequency point per each resonance
  - The electrical length of a system should not change more than quarter of wave-length between two consecutive points

$$df < \frac{c}{4L \cdot \sqrt{\varepsilon_{eff}}}$$



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Summary: Frequency-dependent or local quality metrics for S-parameters

- Reciprocity: $RM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j} - S_{j,i} \right|$
- Passivity: $PM = \sqrt{\max\left[eigenvals(S^* \cdot S) \right]}$ (incomplete without causality verification)
- Geometric Symmetry: $GSM = \frac{1}{N_s} \sum_{i,j} \left| S_{i,j} - S_{is,js} \right|$

Global quality metrics (0-100%)

- Passivity Quality Measure: PQM or zero if PQM<0

$$PQM = \frac{100}{N_{total}} \left[N_{total} - \sum_{n=1}^{N_{total}} PW_n \right] \% \quad PW_n = 0 \; if \; PM_n < 1.00001; \; otherwise \; PW_n = \frac{PM_n - 1.00001}{0.1}$$

- Reciprocity Quality Measure: RQM or zero if RQM<0

$$RQM = \frac{100}{N_{total}} \left[N_{total} - \sum_{n=1}^{N_{total}} RW_n \right] \% \quad RW_n = 0 \; if \; RM_n < 10^{-6}; \; otherwise \; RW_n = \frac{RM_n - 10^{-6}}{0.1}$$

- Geometric Symmetry Quality Measure: SQM or zero if SQM<0

$$SQM = \frac{100}{N_{total}} \left[N_{total} - \sum_{n=1}^{N_{total}} SW_n \right] \% \quad SW_n = 0 \; if \; SM_n < 10^{-6}; \; otherwise \; SW_n = \frac{SM_n - 10^{-6}}{0.1}$$

- Causality Quality Measure: Minimal ratio of clockwise rotation measure to total rotation measure in %
- RMS error of the rational compact model can be also used as the quality or causality measure

Part II: Practical examples

- Teraspeed's PLRD-1 (SOLT&TRL)
- Samtec's connectors and test-boards
- …

Microwave signal integrity software

- ADS from Agilent Technologies
- Ansoft Designer SI
- AWR's SI Design Suite
- Simbeor from Simberian

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- D. F. Williams, L. A. Hayden, R. B. Marks, A complete multimode equivalent-circuit theory for electrical design, Journal of Research of the National Institute of Standards and Technology, v. 102, No. 4, 1997, p. 405-423.
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- G. G. Gentili, M. Salazar-Palma, The definition and computation of modal characteristic impedance in quasi-TEM coupled transmission lines. IEEE Trans. on MTT, v. 43, No. 2, p. 338-343.
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Backup slides

- Linear algebra 101
- Examples of Y,Z and S-parameters for simple multiports
- Generalized theory of multi-conductor transmission lines
- Smoothing data
- RCM building algorithm
- Broad-band SPICE model

Matrices and vectors

- Column-vector is an array with m rows and 1 column
  Example of complex column-vector

$$\overline{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}, \ \overline{c} \in C^{4\times 1} \qquad \overline{c} \nearrow$$

- Row-vector is an array with 1 row a and n columns
  Example of complex row-vector

$$\overline{r} = [r_1, r_2, r_3, r_4], \ \overline{r} \in C^{1\times 4} \qquad \overline{r} \nearrow$$

- m by n matrix is an array with m rows and n columns - for multiport applications the elements are complex numbers or functions of frequency in general
  Example of complex matrix 4 by 4

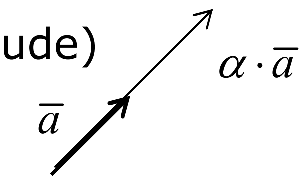
$$A = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix}, \ A \in C^{4\times 4}$$

- Vector norm is the measure of vector magnitude
  Example of Euclidian norm

$$\left\| \overline{x} \right\|_2 = \sqrt{\sum_{i=1}^{N} \left| x_i \right|^2}$$

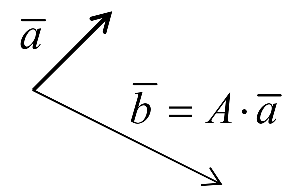
Products

- Vector times a scalar: $\alpha \cdot \bar{a} = \begin{bmatrix} \alpha \cdot a_1 \\ \alpha \cdot a_2 \\ \alpha \cdot a_3 \\ \alpha \cdot a_4 \end{bmatrix}$ (change of magnitude)



- Matrix times a vector: $\bar{b} = A \cdot \bar{a}$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$



is a linear combination of the columns of A:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = a_1 \begin{bmatrix} A_{1,1} \\ A_{2,1} \\ A_{3,1} \\ A_{4,1} \end{bmatrix} + a_2 \begin{bmatrix} A_{1,2} \\ A_{2,2} \\ A_{3,2} \\ A_{4,2} \end{bmatrix} + a_3 \begin{bmatrix} A_{1,3} \\ A_{2,3} \\ A_{3,3} \\ A_{4,3} \end{bmatrix} + a_4 \begin{bmatrix} A_{1,4} \\ A_{2,4} \\ A_{3,4} \\ A_{4,4} \end{bmatrix}$$

- Matrix times matrix $B = A \cdot C$ - each column of B is a linear combination of the columns of A $A \in C^{l \times m}, C \in C^{m \times n} \Rightarrow B \in C^{l \times n}$

4 by 4 example:

$$\begin{bmatrix} \ldots B_{1,k} \ldots \\ \ldots B_{2,k} \ldots \\ \ldots B_{3,k} \ldots \\ \ldots B_{4,k} \ldots \end{bmatrix} = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{bmatrix} \cdot \begin{bmatrix} \ldots C_{1,k} \ldots \\ \ldots C_{2,k} \ldots \\ \ldots C_{3,k} \ldots \\ \ldots C_{4,k} \ldots \end{bmatrix}$$

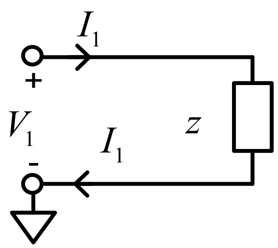
$A, B, C \in C^{4 \times 4}$

More of familiar definitions

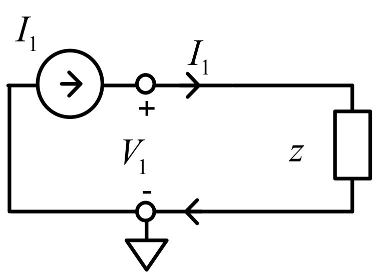
- Linearity: $\overline{b}_1 = A\cdot\overline{a}_1,\ \overline{b}_2 = A\cdot\overline{a}_2\ \Rightarrow \overline{b}_1+\overline{b}_2 = A\cdot\left(\overline{a}_1+\overline{a}_2\right)$
- Two non-zero vectors are linearly independent if
  $\alpha\cdot\overline{a}+\beta\cdot\overline{b}=0\ \ only\ \ with\ \alpha=0\ and\ \beta=0$
- Rank of matrix A is the dimension of the column space of A, or number of linearly independent columns of A
- Range of matrix A is the set of vectors that can be expressed as *Ax* for all *x* from the domain-space
  - The range is the space spanned by the columns of A
- Matrix A can be inverted if it is N by N and rank(A)=N (non-singular or full rank matrix): $\overline{b} = A\cdot\overline{a} \ \Rightarrow\ \overline{a} = A^{-1}\cdot\overline{b}$

  a is vector of coefficients of unique linear expansion of *b* in the basis of columns *A*

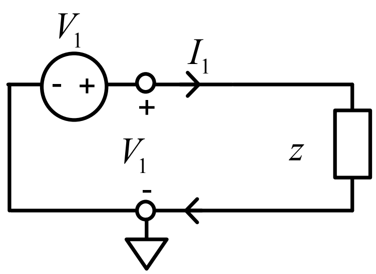
Example: Terminator, one-port



z is a complex impedance



$$Z \in C^{1\times 1},\quad Z_{1,1} = \frac{V_1}{I_1} = z$$



$$Y \in C^{1\times 1},\quad Y_{1,1} = \frac{I_1}{V_1} = \frac{1}{z}$$

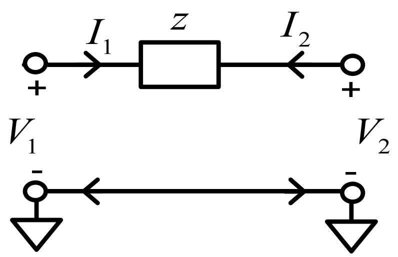
Passivity:

$$\mathrm{Re}(z) \geq 0$$

Always satisfied for nets composed of R,L,C

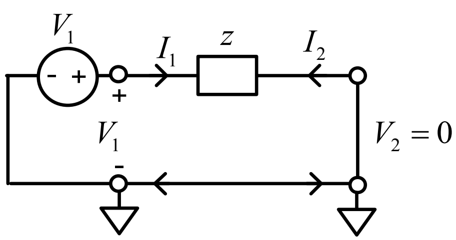
Elements of Y are infinities where z is zero

Example: Series impedance, two-port

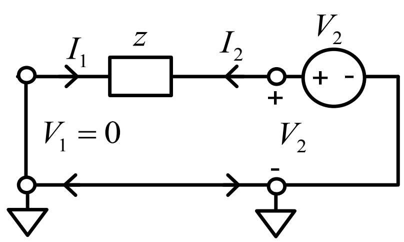


z is a complex impedance

$$Y = \frac{1}{z}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad Y \in C^{2\times 2}$$



$$Y_{1,1} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{z} \qquad Y_{2,1} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{z}$$



$$Y_{2,2} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{z} \qquad Y_{1,2} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{z}$$

Y-matrix is always symmetric (reciprocal)
Y-matrix is singular (column-vectors are linearly-dependent) – no Z-matrix!
Elements of Y are infinities where z is zero

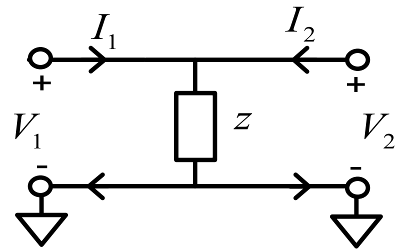
Passivity:

$$eigenvals\left[\mathrm{Re}(Y)\right] = \left\{\mathrm{Re}\left(\frac{2}{z}\right), 0\right\} \geq 0$$

$$\mathrm{Re}(z) \geq 0$$

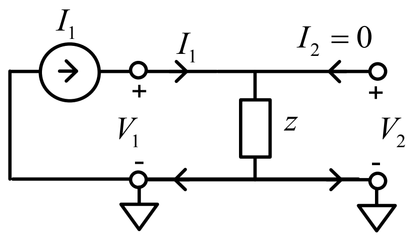
Always satisfied for nets composed of R,L,C

Example: Parallel impedance, two-port

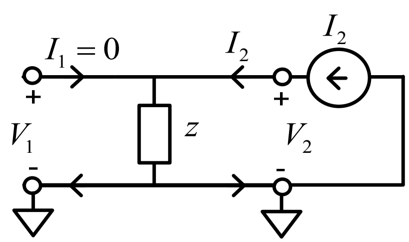


z is a complex impedance

$$Z = z\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad Z \in C^{2\times 2}$$



$$Z_{1,1} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z \qquad Z_{2,1} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z$$



$$Z_{2,2} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z \qquad Z_{1,2} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z$$

Z-matrix is always symmetric (reciprocal)

Z-matrix is singular (column-vectors are linearly-dependent) – no Y-matrix!

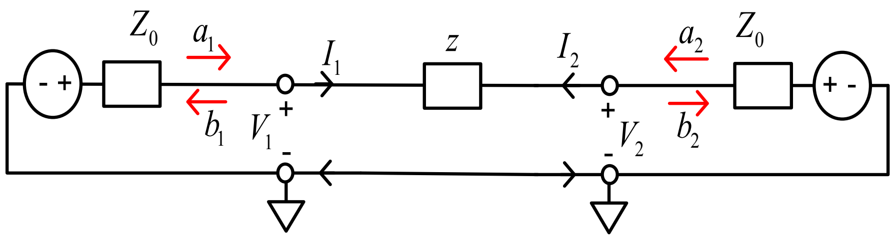
Passivity:

$$eigenvals\left[\mathrm{Re}(Z)\right] = \left\{\mathrm{Re}(2z), 0\right\} \geq 0$$

$$\mathrm{Re}(z) \geq 0$$

Always satisfied for nets composed of R,L,C

Example: Series impedance, two-port



z is a complex impedance $S \in C^{2\times 2}$

We just use known Y and transform it to S

$$Y = \frac{1}{z}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow Y_N = Z_0 \cdot Y = \frac{Z_0}{z}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$S = (U - Y_N)\cdot(U + Y_N)^{-1} = \frac{1}{z + 2\cdot Z_0}\begin{bmatrix} z & 2\cdot Z_0 \\ 2\cdot Z_0 & z \end{bmatrix}$$

Short-circuit:

$$z = 0 \Rightarrow S_{1,1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

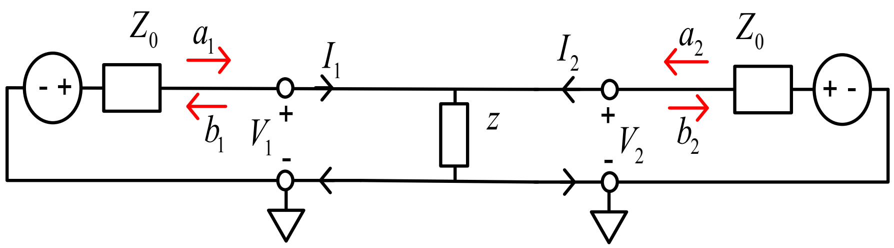
Open-circuit:

$$z = \infty \Rightarrow S_{1,1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Passivity: $$\left|eigenvals[S]\right| = \left\{\left|\frac{z - 2\cdot Z_0}{z + 2\cdot Z_0}\right|, 1\right\} \le 1 \Rightarrow \mathrm{Re}(z) \ge 0$$ For real normalization impedance

S-matrix is always symmetric (reciprocal system) and non-singular for any z

Example: Parallel impedance, two-port



z is a complex impedance

$$S \in C^{2\times 2}$$

We just use known Z and transform it to S

$$Z = z\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow Z_N = \frac{1}{Z_0}\cdot Z = \frac{z}{Z_0}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$S = \left(Z_N - U\right)\cdot\left(U + Z_N\right)^{-1} = \frac{1}{y + 2\cdot Y_0}\begin{bmatrix} -y & 2\cdot Y_0 \\ 2\cdot Y_0 & -y \end{bmatrix}$$

$$y = \frac{1}{z},\quad Y_0 = \frac{1}{Z_0}$$

Short-circuit:

$$z = 0 \Rightarrow S_{1,1} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

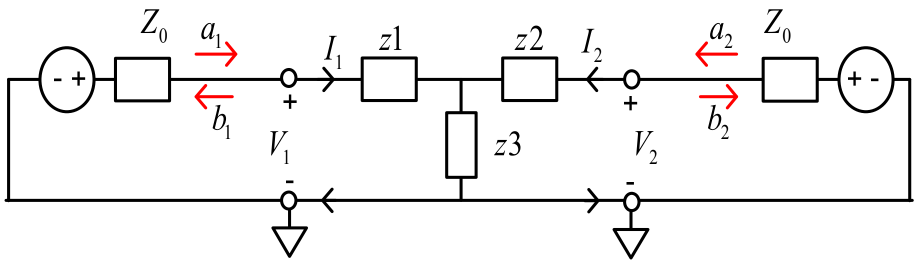
Open-circuit:

$$z = \infty \Rightarrow S_{1,1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Passivity: $\left|eigenvals[S]\right| = \left\{\left|\frac{y - 2\cdot Y_0}{y + 2\cdot Y_0}\right|, 1\right\} \le 1 \Rightarrow \mathrm{Re}(z) \ge 0$ For real normalization impedance

S-matrix is always symmetric (reciprocal system) and non-singular for any z

Example: Ideal 3-dB attenuator



z1=z2=8.56 Ohm,
z3=141.8 Ohm
Zo=50 Ohm

$$S = \frac{1}{A}\begin{bmatrix} -Z_0^2 + (z1 - z2)\cdot Z_0 + B & 2\cdot z3\cdot Z_0 \\ 2\cdot z3\cdot Z_0 & -Z_0^2 - (z1 - z2)\cdot Z_0 + B \end{bmatrix} = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

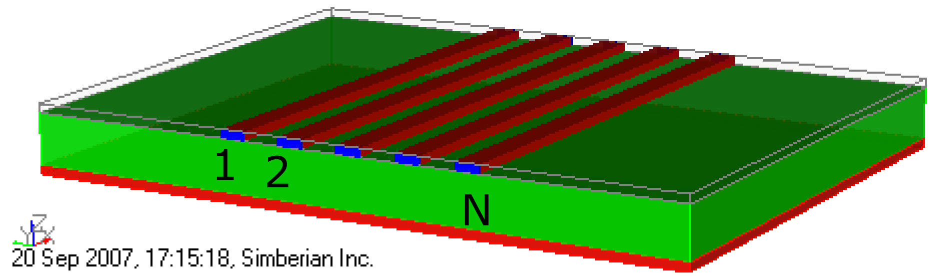
$$A = Z_0^2 + (z1 + z2 + 2\cdot z3)\cdot Z_0 + B = 20036 \qquad B = z1\cdot z2 + z2\cdot z3 + z1\cdot z3 = 2500$$

$$\left|S_{1,2}\right|_{dB} = 20\cdot\log\left(\left|S_{1,2}\right|\right) \approx 3\,dB$$

Transmission line description with generalized Telegrapher's equations

$$\frac{\partial \bar{V}(x)}{\partial x} = -Z(\omega)\cdot \bar{I}(x)$$

$$\frac{\partial \bar{I}(x)}{\partial x} = -Y(\omega)\cdot \bar{V}(x)$$



Plus boundary conditions at the ends of the segment

$$Z(\omega) = R(\omega) + i\omega\cdot L(\omega)$$

$$Y(\omega) = G(\omega) + i\omega\cdot C(\omega)$$

I – complex column-vector of N currents

V – complex column-vector of N voltages

Z [Ohm/m] and *Y* [S/m] are complex NxN matrices of impedances and admittances per unit length

R [Ohm/m], L [Hn/m] – real NxN frequency-dependent matrices of resistance and inductance per unit length

G [S/m], C [F/m] – real NxN frequency-dependent matrices of conductance and capacitance per unit length

Transformation to modal space

$$Z(\omega) = R(\omega) + i\omega \cdot L(\omega)$$
$$Y(\omega) = G(\omega) + i\omega \cdot C(\omega)$$

Per unit length matrix parameters (NxN complex matrices)

Symmetric in case of linear isotropic materials

$$Z(\omega) \cdot Y(\omega) \cdot M_V = \Lambda \cdot M_V$$
$$Y(\omega) \cdot Z(\omega) \cdot M_I = \Lambda \cdot M_I$$

Eigen-vectors of ZY and YZ are actually the modes of the line – they can be used to form current M_I and voltage M_V transformation matrices

$$\overline{V} = M_V \cdot \overline{v}$$
$$\overline{I} = M_I \cdot \overline{i}$$

Definition of terminal voltage and current vectors through modal voltage and current vectors and modal transformation matrices

$$y(\omega) = M_I^{-1} \cdot Y(\omega) \cdot M_V$$
$$z(\omega) = M_V^{-1} \cdot Z(\omega) \cdot M_I$$

Matrices of impedances and admittances per unit length are transformed into **diagonal form** with the current M_I and voltage M_V transformation matrices (both are frequency-dependent)

$$W = M_I^t \cdot M_V = M_V^t \cdot M_I$$

Matrix W is diagonal for the reciprocal systems because of Z and Y must stay symmetric during the modal tranformation

$$P = M_V^t \cdot M_I^*$$

Complex power transferred along the line (may be fully populated)

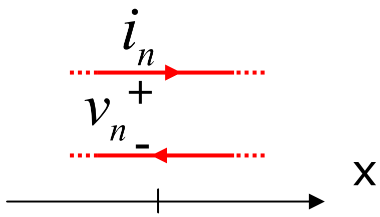
Waves in multi-conductor t-lines

$$Z_{0n}(\omega) = \sqrt{z_{n,n}(\omega) / y_{n,n}(\omega)}$$

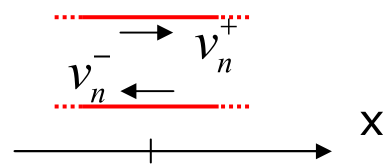
$$\Gamma_n(\omega) = \sqrt{z_{n,n}(\omega) \cdot y_{n,n}(\omega)}$$

Modal complex characteristic impedance and propagation constant

Current and voltage of mode number n (n=1,…,N)



Voltage waves for mode number n (n=1,…,N)



$$v_n(x) = v_n^+ \cdot \exp(-\Gamma_n \cdot x) + v_n^- \cdot \exp(\Gamma_n \cdot x)$$

$$i_n(x) = \frac{1}{Z_{0n}} \left[v_n^+ \cdot \exp(-\Gamma_n \cdot x) - v_n^- \cdot \exp(\Gamma_n \cdot x) \right]$$

$$P_n^+ = \frac{|v_n^+|^2}{Z_{0n}}$$

$$P_n^- = \frac{|v_n^-|^2}{Z_{0n}}$$

Passivity:

$$\mathrm{Re}(Z_{0n}(\omega)) \geq 0$$

$$\alpha_n = \mathrm{Re}(\Gamma_n(\omega)) \geq 0$$

$$\overline{V} = M_V \cdot \overline{v}$$
$$\overline{I} = M_I \cdot \overline{i}$$

Voltage and current in multiconductor line can be expressed as a superposition of modal currents and voltages

One and two-conductor lines

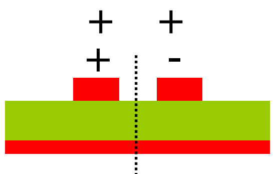
One-conductor case

$$M_V = M_I = 1$$

$$Z_0(\omega) = \sqrt{Z(\omega)/Y(\omega)}$$

$$\Gamma(\omega) = \sqrt{Z(\omega)\cdot Y(\omega)}$$

Symmetric two-conductor case – even and odd mode normalization



$$M_V = M_I = M_{eo} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$y_{eo} = M_{eo}\cdot Y(\omega)\cdot M_{eo}$$

$$z_{eo} = M_{eo}\cdot Z(\omega)\cdot M_{eo}$$

$$Z_{odd}(\omega) = \sqrt{z_{eo1,1}/y_{eo1,1}} \qquad \Gamma_{odd}(\omega) = \sqrt{z_{eo2,2}\cdot y_{eo2,2}}$$

$$Z_{even}(\omega) = \sqrt{z_{eo2,2}/y_{eo2,2}} \qquad \Gamma_{even}(\omega) = \sqrt{z_{eo2,2}\cdot y_{eo2,2}}$$

Common and differential mode normalization

$$M_V = M_{Vmm} = \begin{bmatrix} 1 & 0.5 \\ -1 & 0.5 \end{bmatrix}, \quad M_I = M_{Imm} = \begin{bmatrix} 0.5 & 1 \\ -0.5 & 1 \end{bmatrix}$$

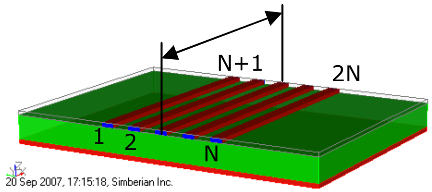
$$y_{mm} = M_{Imm}^{-1}\cdot Y(\omega)\cdot M_{Vmm}$$

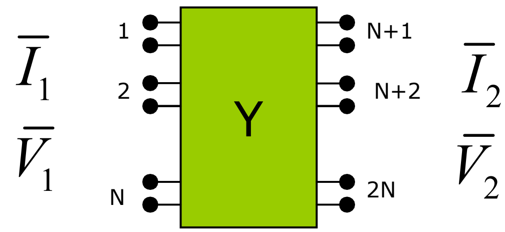
$$z_{mm} = M_{Vmm}^{-1}\cdot Z(\omega)\cdot M_{Imm}$$

$$Z_{differential}(\omega) = \sqrt{z_{mm1,1}/y_{mm1,1}} \qquad Z_{differential} = 2\cdot Z_{odd} \qquad \Gamma_{differential} = \Gamma_{odd}$$

$$Z_{common}(\omega) = \sqrt{z_{mm2,2}/y_{mm2,2}} \qquad Z_{common} = 0.5\cdot Z_{even} \qquad \Gamma_{common} = \Gamma_{even}$$

Admittance parameters of multiconductor line segment





$$\tilde{Y}(\omega,l)=\begin{bmatrix} diag\left(\frac{cth(\Gamma_n l)}{Z_{0n}}\right) & diag\left(-\frac{csh(\Gamma_n l)}{Z_{0n}}\right) \\ diag\left(-\frac{csh(\Gamma_n l)}{Z_{0n}}\right) & diag\left(\frac{cth(\Gamma_n l)}{Z_{0n}}\right) \end{bmatrix}$$

2N x 2N three-diagonal admittance matrix of the line segment in the modal space

$$Y(\omega,l)=\begin{bmatrix} M_I & 0 \\ 0 & M_I \end{bmatrix}\cdot\tilde{Y}(\omega,l)\cdot\begin{bmatrix} M_V^{-1} & 0 \\ 0 & M_V^{-1} \end{bmatrix}$$

2N x 2N admittance matrix of the line segment in the terminal space – symmetric in case of reciprocal system $W = M_I^t \cdot M_V = M_V^t \cdot M_I$

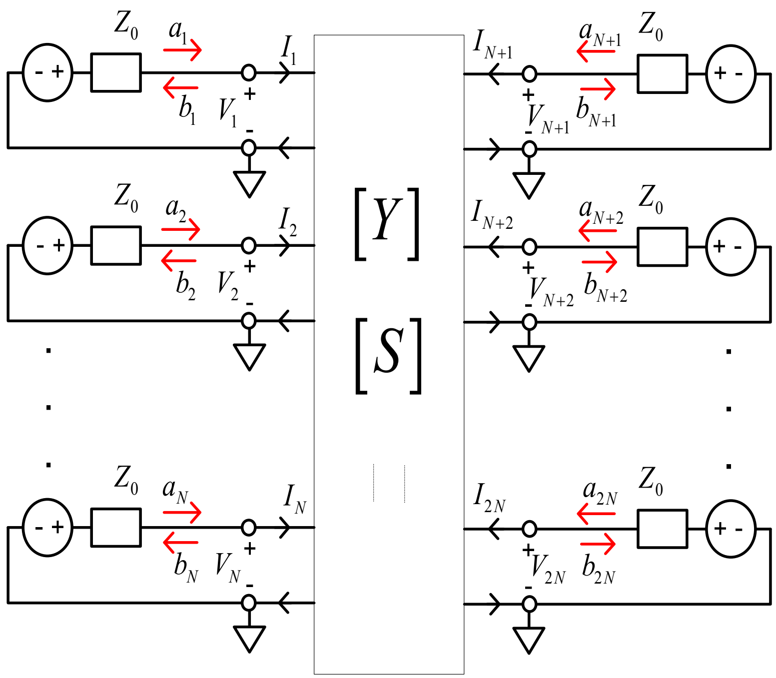
$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = Y(\omega,l)\cdot\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

Admittance matrix leads to a system of linear equations with voltages and currents at the external line terminals

The most general passivity condition (reciprocal system): $eigenvals\left[\mathrm{Re}\left(Y(\omega,l)\right)\right]\geq 0$

To have real-positive characteristic impedances and propagation constants: $eigenvals\left[\mathrm{Re}\left(Z(\omega)\right)\right]\geq 0,\ eigenvals\left[\mathrm{Re}\left(Y(\omega)\right)\right]\geq 0$

Scattering parameters of multiconductor line segment



$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = Y(\omega,l) \cdot \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

Admittance parameters (known)

$$\bar{a}_{1,2} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V}_{1,2} + Z_0 \cdot \bar{I}_{1,2} \right)$$

Vectors of incident waves

$$\bar{b}_{1,2} = \frac{1}{2} Z_0^{-1/2} \cdot \left(\bar{V}_{1,2} - Z_0 \cdot \bar{I}_{1,2} \right)$$

Vectors of reflected waves

$$\begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \end{bmatrix} = S(\omega,l) \cdot \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}$$

Scattering parameters

$$S \in C^{2N \times 2N}$$

$$Y_N = Z_0^{1/2} \cdot Y(\omega,l) \cdot Z_0^{1/2}$$

Normalization matrix is diagonal matrix with normalization impedances on the diagonal

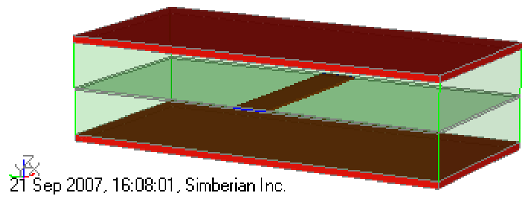
$$S(\omega,l) = (U - Y_N) \cdot (U + Y_N)^{-1}$$

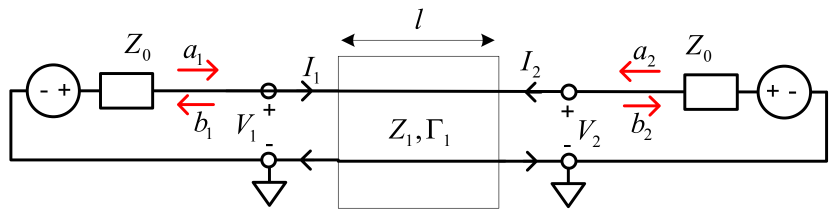
S-matrix of the line segment is computed as the Cayley transform of the normalized Y-matrix

Simple strip-line segment example

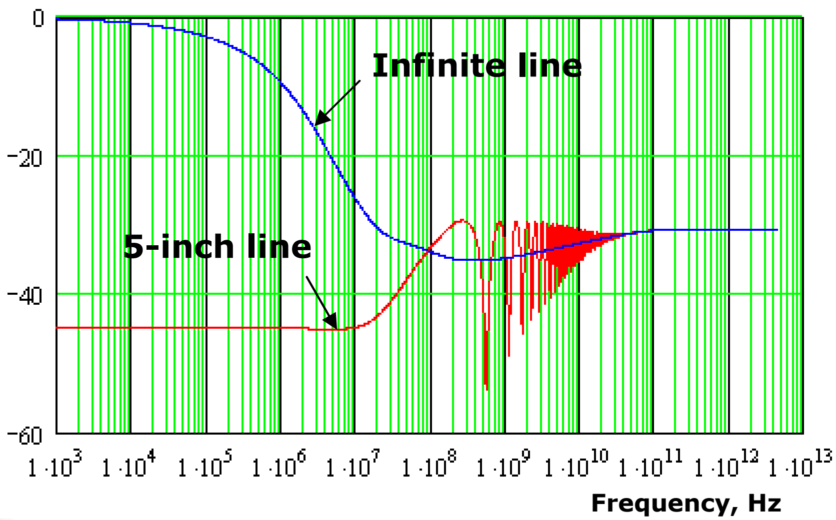
8-mil strip, 20-mil plane to plane distance, DK=4.2, LT=0.02 at 1 GHz, wide-band Debye dielectric model.

Strip is made of copper, planes are ideal, no roughness, no high-frequency dispersion.

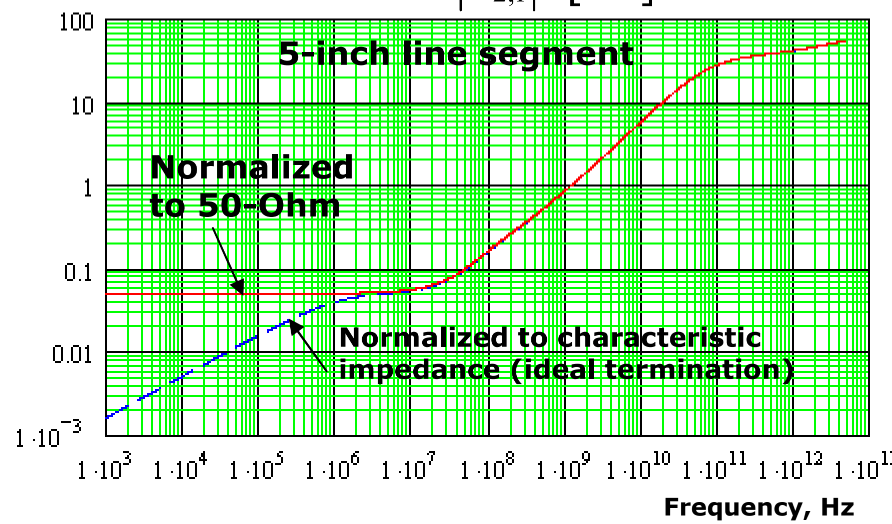




$$20 \log |S_{1,1}|, [dB]$$

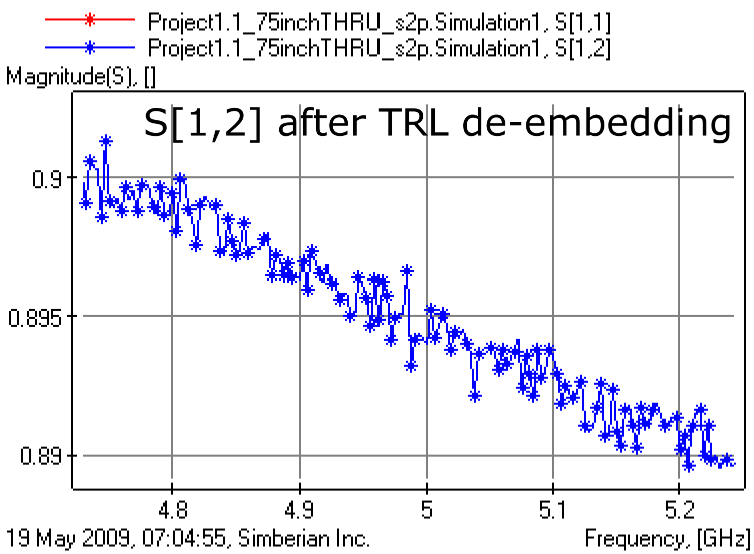


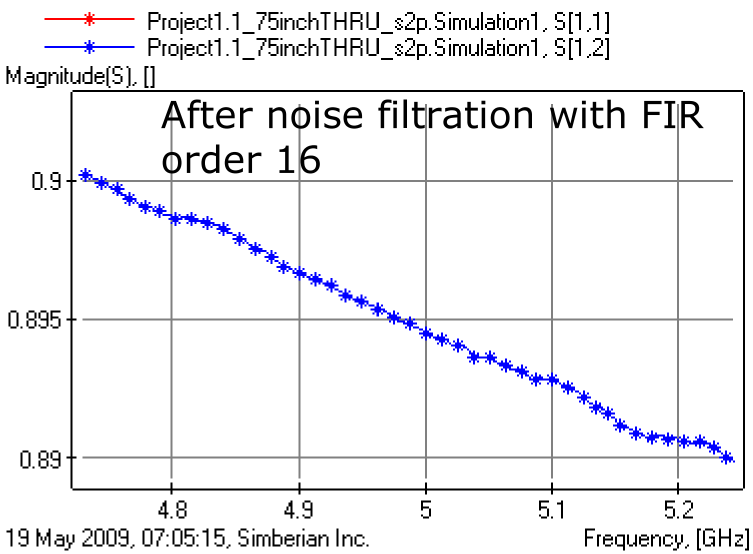
$$-20 \log |S_{2,1}|, [dB]$$



Smoothing data with FIR filter

- Moving average FIR (boxcar) filter is used to smooth the data
- It may be helpful in case of noisy oversampled data (small phase difference between consecutive frequency points)
- Cannot be used if data are just sufficiently sampled or under-sampled
- Example of data before and after filtration:





How RCMs are constructed?

1. Delay is extracted if possible
2. Number of poles and their locations are guessed
3. Linear system is constructed from the rational approximation:

$$H(s)=\frac{d+h\cdot s+\sum_{n=1}^{N_p}\frac{c_n}{s-p_n}-\frac{c_n^*}{s-p_n^*}}{1+\sum_{n=1}^{N_p}\frac{\tilde{c}_n}{s-p_n}-\frac{\tilde{c}_n^*}{s-p_n^*}}$$

$s=i\omega-frequency,\ d-value\,at\,\infty,$
$h-asymptote\,(zero\ \ for\ S-parameters),$
$N_p-number\ of\ poles,$
$p_n-poles\,(real\ or\ complex),$
$c_n-residues\,(unknown),$
$\tilde{c}_n-auxiliary\ residues\,(unknown)$

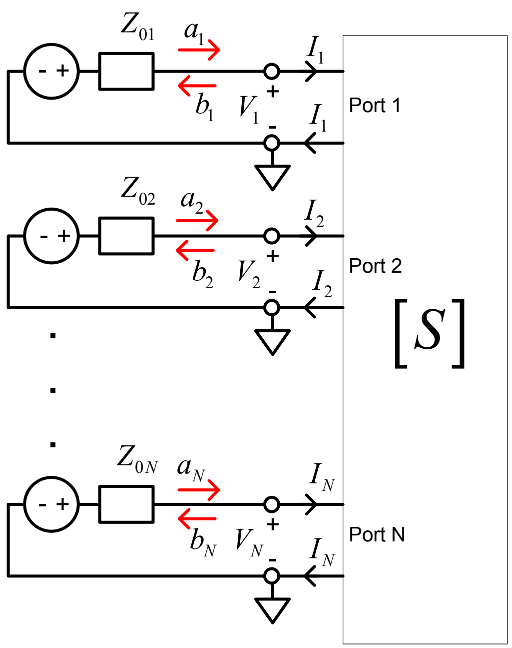
4. The linear system size is 2*Ns x 2Np+2 – solved with LQ-decomposition
   2Np+2 unknown residues require Ns number of samples Ns>=2Np+2 assuming all poles are complex
5. Zeroes of the auxiliary function become new poles (only with negative real part)

$$\sigma(s)=1+\sum_{n=1}^{N_p}\frac{\tilde{c}_n}{s-p_n}-\frac{\tilde{c}_n^*}{s-p_n^*}$$

6. Residues are found solving the system $\Rightarrow$ $H(s)=d+h\cdot s+\sum_{n=1}^{N_p}\left(\frac{r_n}{s-p_n}+\frac{r_n^*}{s-p_n^*}\right)$
7. Items 3-6 are repeated until convergence criterion is satisfied (soft relocation)

Details of the vectfit (or Sanathanan-Koerner) procedure are in B. Gustavsen, A. Semlien, Rational approximation of frequency domain responses by vector fitting, IEEE Trans. on Power Delivery, v. 14, 1999, N3, p. 1052-1061.

Converting RCM into a SPICE model



$$\bar{b} = S \cdot \bar{a}, \quad S_{i,j} = \left.\frac{b_i}{a_j}\right|_{a_k=0\ k\neq j}$$ ⇦ scattering matrix

$$\bar{a} = \frac{1}{2\sqrt{Z_0}}\left(\bar{V} + Z_0 \cdot \bar{I}\right), \quad \bar{b} = \frac{1}{2\sqrt{Z_0}}\left(\bar{V} - Z_0 \cdot \bar{I}\right)$$ ⇦ waves

can be also treated as:

$$b_i = \frac{1}{2\sqrt{Z_{0i}}} V_i - \frac{\sqrt{Z_{0i}}}{2} \cdot I_i \Rightarrow V_i = Z_{0i} \cdot I_i + 2\sqrt{Z_{0i}} \cdot b_i$$ ⇦ b_i as current

$$a_i = \frac{1}{2\sqrt{Z_{0i}}} V_i + \frac{\sqrt{Z_{0i}}}{2} \cdot I_i, \quad b_i = \sum_{j=1}^{N} S_{i,j} \cdot a_j$$ ⇦ a_j as voltage

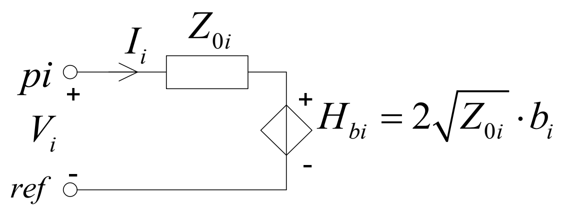
$$S_{i,j} = \left[d_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{s - p_{ij,n}} + \frac{r^*_{ij,n}}{s - p^*_{ij,n}} \right) \right] \cdot e^{-s \cdot T_{ij}}$$ ⇦ S_{ij} as voltage-controlled current sources

Simple and LAPLACE VCCS can be used to represent each element of the S-matrix rational approximation with VCVS for the delay element

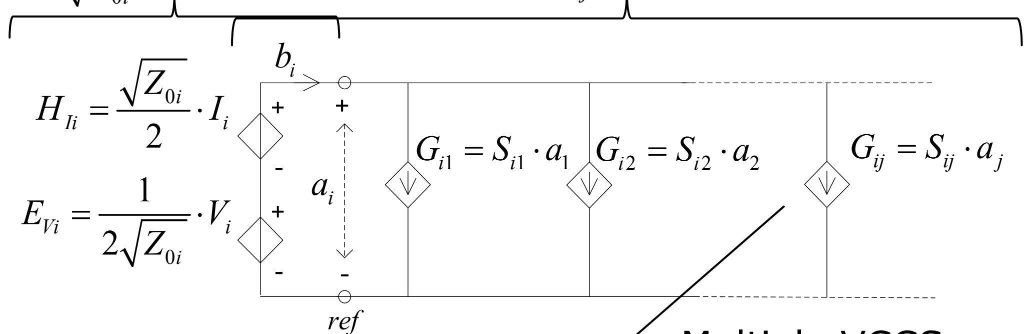
First published in: J. De Geest, S. Sercu, C. Clewell, J. Nadolny, Making S-parameters suitable for SPICE modeling, - DesignCon2004.
Also in: N. Stevens, T. Dhaene, Generation of rational model based SPICE circuits for transient simulations, - SPI2008.

Broad-band SPICE circuit

$$V_i = Z_{0i} \cdot I_i + 2\sqrt{Z_{0i}} \cdot b_i$$



$$a_i = \frac{1}{2\sqrt{Z_{0i}}} V_i + \frac{\sqrt{Z_{0i}}}{2} \cdot I_i \qquad b_i = \sum_{j=1}^{N} S_{i,j} \cdot a_j$$

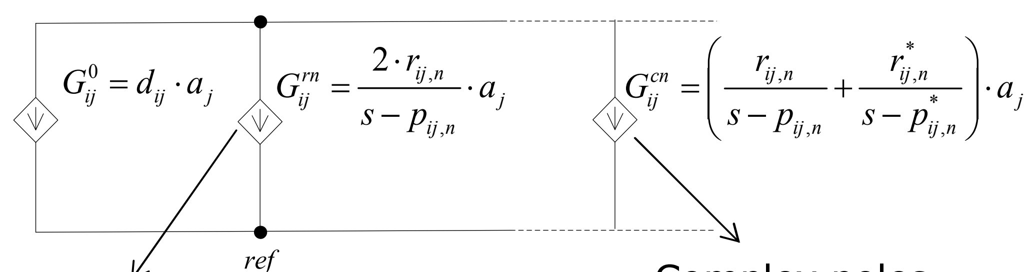


Multiple VCCSs in parallel

Just circuit for a port number i is shown here – similar circuits are used for all ports of a multiport

$$S_{i,j} = d_{ij} + \sum_{n=1}^{N_{ij}} \left(\frac{r_{ij,n}}{s - p_{ij,n}} + \frac{r^*_{ij,n}}{s - p^*_{ij,n}} \right)$$

Delay element is not shown here – it is implemented as VCVS with delay loaded by a dummy resistor



Real poles

Complex poles